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Standards-Based Mathematics 7

Help Pages

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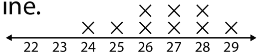
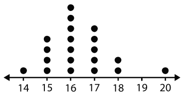
Help Pages

Vocabulary

absolute deviation	a measure of variability; in a set of data the absolute difference between a data point and another point, such as the mean or median. Example: if the median is 3 and a data point is 5 its absolute deviation from the median is 2 because the difference between 3 and 5 is 2. Absolute deviation is never negative (*see* absolute value).
absolute value	the distance between a number and zero on a number line. Example: the absolute value of negative seven is 7; it is written as $\lvert -7 \rvert$. Absolute value is never negative.
acute angle	an angle measuring less than 90°
additive inverse property	a math rule that states that the sum of any number and its inverse is zero. $-5 + 5 = 0$ is an example of the additive inverse property.
adjacent angles	angles that share a common vertex and a common side, but do not overlap. Adjacent angles are created when two lines intersect. The angles are directly next to one another.
algebraic expression	a mathematical phrase written in numbers and symbols. Example: $2x + 5$.
approximately symmetric	a distribution that appears to be a mirror reflection above and below the median; A bell curve is an example; the word *approximately* means "fairly close," so the bell curve may not be perfectly symmetric but is close to symmetric.
area	the amount of space within a polygon; area is always measured in square units (feet^2, meters^2, etc.)
associative property	a math rule which states that changing the grouping of addends or factors in an equation does not change the outcome. Example: $(a + b) + c = a + (b + c)$ or $(a \times b) \times c = a \times (b \times c)$
axis / axes	the lines that form the framework for a graph. The horizontal axis is called the x-axis; the vertical axis is called the y-axis.
chord	any line segment that touches two points along the circumference of a circle.
circumference	the distance around the outside of a circle
cluster	numbers that are bunched or grouped together in a set of values
coefficient	a number in front of a variable in an algebraic term. Example: in the term 5x, 5 is the coefficient.
commutative property	a math rule which states that changing the order of addends or factors in an equation does not change the outcome. Example: $a + b = b + a$ and $a \times b = b \times a$
complementary angles	two angles whose sum is 90°
complex fraction	fractions that have fractions in the numerator and/or fractions in the denominator Examples: $\frac{\frac{2}{3}}{6}$ $\frac{\frac{3}{7}}{\frac{1}{5}}$
congruent	figures with the same shape and the same size
constant	in an algebraic expression, a number that is not attached to a variable; a term that always has the same value. Example: In the expression $3x + 4$, the number 4 is a constant.
constant of proportionality	the numerical portion of a unit rate
coordinates	an ordered pair of numbers that give the location of a point on a coordinate grid
coordinate plane/grid	a grid in which the location is described by its distances from two intersecting, straight lines called axes
dependent variable	a variable that is affected by the independent variable. In $y = 3x$, the value of y depends on what is done to x.
diameter	a special chord that runs through the center of a circle.

Help Pages

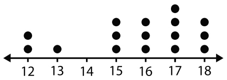
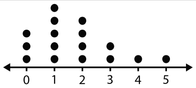
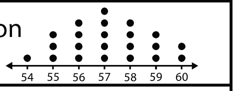
Vocabulary	
distributive property	a math rule that is applied when one term is multiplied by an expression that includes either addition or subtraction. Example: $a(x + y) = ax + ay$
dot plot	a graphic that summarizes a set of data. (*see* line plot)
evaluate	to find the value of an expression
exponent	tells the number of times that a base is multiplied by itself. An exponent is written to the upper right of the base. Example: $5^3 = 5 \times 5 \times 5$. The exponent is 3.
exponential notation	an expression with an exponent. 4^3 is an example of an exponential notation.
face	a flat surface of a solid figure
fundamental counting principle	a way to determine the possible outcomes or combinations for a series of choices
gap	a large space between data or missing data from an established set of values
greatest common factor (GCF)	the highest factor that 2 numbers have in common. Example: The factors of 6 are 1, 2, 3, and 6. The factors of 9 are 1, 3, and 9. The GCF of 6 and 9 is 3.
independent variable	a variable that affects the dependent variable. For example, the independent variable is x in $y = 3x$. In this expression, when x is 2, $y = 6$; when x is 0.5, $y = 1.5$.
inequality	a statement that one quantity is different than another
integers	the set of whole numbers, positive, negative, and zero. A set of integers includes zero, the counting numbers, and their opposites.
interest	amount paid or earned for the use of money over time
interest rate	percentage charged or paid for the use of money over time
interquartile range	a measure of variability; the range of the middle 50% of a data set; IQR is the difference between the upper and lower quartiles (Q3 – Q1)
like terms	terms that have the same variable and are raised to the same power; like terms can be combined (added or subtracted), whereas unlike terms cannot
line plot	a graphic that summarizes a set of data. Each data point is represented by a mark or dot above a number line.  
linear expression	an algebraic expression in which the variable is raised to the first power. It is sometimes simply called an expression. $2x$ and $6x + 4$ are examples of linear expressions.
mean	a measure of center; the average of a set of numbers
mean absolute deviation	in a data set, the average of the differences between each data point and the mean of the set; mean absolute deviation (MAD) indicates the degree of variability in a data set
median	a measure of center; the data point that falls in the exact middle of a set of ordered data
measure of center	mean, median, mode; a number that summarizes a set of data
measure of variability	a number that indicates the degree of variance (how clustered or spread out a set of data is). Examples are range, interquartile range, and mean absolute deviation.
minimum	the smallest number; the lowest value in a data set
mode	a measure of center; in a data set, the value that occurs most often

Help Pages

Vocabulary

negative numbers	all the numbers less than zero. (Zero is neither positive nor negative.) A negative number has a negative sign (–) in front of it.
obtuse angle	an angle measuring more than 90°
order of operations	a rule that tells the order in which to perform operations in an equation. Solve whatever is in parenthesis or brackets first (work from the innermost grouping outward); then calculate exponents, then do multiplication or division (from left to right), and finally, addition or subtraction (from left to right).
ordered pair	a pair of numbers that gives the coordinates of a point on a grid
origin	the point where the *x*-axis and *y*-axis intersect, the point (0, 0)
opposite numbers	two numbers that are exactly the same distance from zero on a number line. Every positive number has an opposite that is negative, and every negative number has an opposite that is positive. Example: 5 and –5 are opposites.
opposite of opposite	the number itself. Examples: The opposite of the opposite of 4 is 4 and the opposite of the opposite of –3 is –3.
outlier	in a set of data, a number that is much smaller or much larger than the other numbers in the set
percent	the ratio of any number to 100; the symbol for percent is % Example: 14% means 14 out of 100 or $\frac{14}{100}$.
pi (π)	the ratio of circumference to diameter of a circle. pi is approximately 3.14, or $\frac{22}{7}$.
population	an entire group. Example: all the students in school, or the citizens of a country
positive numbers	all numbers greater than zero; sometimes a positive sign (+) is written in front of a positive number.
principal	amount of money borrowed or invested
prism	a three-dimensional figure that has two identical, parallel bases and three or more rectangular faces
proportion	a statement that two ratios (or fractions) are equal. Example: $\frac{1}{2} = \frac{3}{6}$
proportional relationship	ratios that are equivalent to each other
pyramid	a three-dimensional figure with one base, which is a polygon (rectangle, triangle, pentagon, hexagon, etc.) and whose faces are all triangles
Q1	the lower quartile; in a box plot, Q1 represents the median of the lower half of the data set.
Q2	the middle quartile; in a box plot, Q2 represents the median of the data set.
Q3	the upper quartile; in a box plot, Q3 represents the median of the upper half of the data set.
quartiles	points that divide a data set into four equal parts or quarters, Q1, Q2, and Q3; see explanation for box and whisker plot
radius	the distance from any point on the circle to the center. The radius is half of the diameter.
range	a measure of variability; in a set of data, the difference between the minimum and maximum values
regular polygon	has sides of equal length and angles that are congruent

Help Pages

Vocabulary	
representative sample	accurately reflects a population and allows a study of the population to be valid
right angle	an angle measuring exactly 90°
sample	a smaller group within a population
sample space	the set of possible outcomes of a probability event
scale	the relationship between two lengths
scale factor	the ratio of the corresponding side lengths of two similar shapes
shape of data	the appearance of a set of data on a dot plot; data is either symmetric or skewed
simulation	a hands-on procedure to understand concepts of probability
skewed	describes a data set that is not evenly balanced; values appear to be pulled toward the right or left. Also, outliers cause data to be skewed
skewed left	data points are more clustered on the right with a "tail" stretching left 
skewed right	data points are more clustered on the left with a "tail" stretching right 
similar	figures having the same shape but different sizes
straight angle	an angle measuring exactly 180°
supplementary angles	two angles whose sum is 180°
surface area	the sum of the areas of the faces of a solid figure. Example: The surface area of a rectangular prism is the sum of the areas of its six faces.
symmetric	a distribution that is evenly balanced and appears to be a mirror reflection above and below the median; a "bell curve" is an example of this 
term	a part of an algebraic expression; a number, variable, or combination of the two; terms are separated by signs, such as +, =, or –
undefined	when 0 is in the denominator of a fraction. For example: $\frac{6}{0}$ or $6 \div 0 \neq 0$ because $0 \times 0 \neq 6$
unit rate	a ratio of two values; in a unit rate, the number in the denominator is 1
variability	the degree to which a set of data is spread out
variable	an unknown or a symbol that stands for an unknown value; a variable can change. In an algebraic expression one must define a variable. This means to choose a letter or symbol to stand for an unknown value.
volume	the number of cubic units it takes to fill a solid; volume is expressed in cubic units Examples: ft^3, m^3, in.^{3}

Help Pages

Expressions and Equations 7.EE.1 – 7.EE.4

Simplifying Algebraic Expressions

Expressions that contain like terms can be simplified. **Like terms** are those that contain the same variable to the same power. $2x$ and $-4x$ are like terms; $3n^2$ and $8n^2$ are like terms; $5y$ and y are like terms; 3 and 7 are like terms.

An expression sometimes begins with like terms. This process for **simplifying expressions** is called **combining like terms**. When combining like terms, first identify the like terms. Then, simply add the like terms to each other and write the results together to form a new expression.

Example: Simplify $2x + 5y - 9 + 5x - 3y - 2$.

The like terms are $2x$ and $+5x$, $+5y$ and $-3y$, and -9 and -2.
$2x + +5x = +7x$, $+5y + -3y = +2y$, and $-9 + -2 = -11$.
The result is $7x + 2y - 11$.

Example: Use the distributive property, and simplify $2(4a + 2b + 6) + 2a$.

Distribute: $8a + 4b + 12 + 2a$
Combine like terms: $8a + 2a + 4b + 12$
Simplify: $10a + 4b + 12$

The next examples are a bit more complex. It is necessary to use the distributive property first, and then to combine like terms.

1. First, apply the distributive property to each expression. Write the results on top of each other, lining up the like terms with each other. Pay attention to the signs of the terms.
2. Then, add each group of like terms. Remember to follow the rules for integers.

Example 1: Simplify. $2(3x + 2y + 2) + 3(2x + 3y + 2)$

Example 2: Simplify. $4(3x - 5y - 4) - 2(3x - 3y + 2)$

Example 1

$$\begin{array}{r} 2\overbrace{(3x+2y+2)}+3\overbrace{(2x+3y+2)} \\ 6x+4y+4 \\ \underline{+6x+9y+6} \\ 12x+13y+10 \end{array}$$

Example 2

$$\begin{array}{r} 4(3x-5y-4)-2(3x-3y+2) \\ +12x-20y-16 \\ \underline{-6x+6y-4} \\ 6x-14y-20 \end{array}$$

An **equation consists of two expressions separated by an equal sign.** You have worked with simple equations for a long time: $2 + 3 = 5$. More complicated equations involve variables that replace a number. To solve an equation like this, you must figure out which number the variable stands for. A simple example is when $2 + x = 5$, $x = 3$. Here, the variable, x, stands for 3.

Sometimes an equation is not so simple. In these cases, there is a process for solving for the variable. No matter how complicated the equation, <u>the goal is to work with the equation until all the numbers are on one side and the variable is alone on the other side</u>. These equations will require only **one step** to solve. To check your answer, put the value of x back into the original equation.

Solving an **equation with a variable on one side:**

1. Look at the side of the equation that has the variable on it. If there is a number added to or subtracted from the variable, it must be removed. In the first example, 13 is added to x.
2. To remove 13, add its opposite (–13) to both sides of the equation.
3. Add downward. x plus nothing is x. 13 plus –13 is zero. 27 plus –13 is 14.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of x. $x = 14$.

Example 1

❶ → $x + 13 = 27$
$\underline{-13 = -13}$ ← ❷
❹ → $x = 14$

Check: 14 + 13 = 27 ✓

Example 2

❶ → $a - 22 = -53$
$\underline{+22 = +22}$ ← ❷
❹ → $a = -31$

Check: −31− 22 = −53 ✓

Help Pages

Expressions and Equations 7.EE.1 – 7.EE.4

Simplifying Algebraic Expressions (continued)

In the next examples, a number is either multiplied or divided by the variable (not added or subtracted).

1. Look at the side of the equation that has the variable on it. If there is a number multiplied by or divided into the variable, it must be removed. In the first example, 3 is multiplied by *x*.
2. To remove 3, divide both sides by 3. You divide because it is the opposite operation from the one in the equation (multiplication).
3. Follow the rules for multiplying or dividing integers. 3*x* divided by 3 is *x*. 39 divided by 3 is 13.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of *x*. *x* = 13.

Example 1: Solve for *x*.

❶ → $3x = 39$

$\frac{3x}{3} = \frac{39}{3}$ ← ❷

❹ → $x = 13$

Check: $3(13) = 39$

$39 = 39$ ✓

Example 2: Solve for *n*.

❶ → $\frac{n}{6} = -15$

$\frac{n}{6}(6) = -15(6)$ ← ❷

❹ → $n = -90$

Check: $\frac{-90}{6} = -15$

$-15 = -15$ ✓

The next set of examples also have a variable on only one side of the equation. These, however, are a bit more complicated, because they will require two steps in order to get the variable alone.

1. Look at the side of the equation that has the variable on it. There is a number (2) multiplied by the variable, and there is a number added to it (5). Both of these must be removed. Always begin with the addition/subtraction. To remove the 5 we must add its opposite (–5) to both sides.
2. To remove the 2, divide both sides by 2. You divide because it is the opposite operation from the one in the equation (multiplication).
3. Follow the rules for multiplying or dividing integers. 2*x* divided by 2 is *x*. 8 divided by 2 is 4.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of *x*. *x* = 4.

Example 3: Solve for *x*.

$2x + 5 = 13$

❶ → $\underline{-5 = -5}$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ ← ❷

❹ → $x = 4$

Check: $2(4) + 5 = 13$

$8 + 5 = 13$

$13 = 13$ ✓

Example 4: Solve for *n*.

$3n - 7 = 32$

❶ → $+7 = +7$

$3n = 39$

$\frac{3n}{3} = \frac{39}{3}$ ← ❷

❹ → $n = 13$

Check: $3(13) - 7 = 32$

$39 - 7 = 32$

$32 = 32$ ✓

This multi-step equation also has a variable on only one side. To get the variable alone, though, requires several steps.

Example: Solve for *x*. 3(2*x* + 3) = 21

1. Use the distributive property.
2. Use the directions for examples 3 and 4 to complete the problem.

$$\begin{aligned} 3(2x + 3) &= 21 \\ 6x + 9 &= 21 \\ \underline{\quad -9} &\underline{= -9} \\ 6x &= 12 \\ \frac{6x}{6} &= \frac{12}{6} \\ x &= 2 \end{aligned}$$

Check: $3(2(2) + 3) = 21$

$3(4 + 3) = 21$

$3(7) = 21$

$21 = 21$ ✓

Help Pages

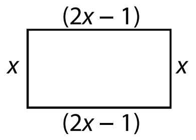
Expressions and Equations 7.EE.1 – 7.EE.4

Finding the Perimeter of a Rectangle

The distance around the outside of a rectangle is the perimeter. A rectangle has 2 pairs of parallel sides. To find the perimeter of a rectangle, add the lengths of the sides, or combine like terms.

Example: If $x = 6$, find the perimeter of this rectangle.

1. Substitute the length and width into the perimeter equation.
2. Simplify.
3. Substitute 6 for x and solve.



$P = l + l + w + w$

$P = (2x - 1) + (2x - 1) + x + x$

$P = 2x + 2(2x - 1) = 2x + 4x - 2$

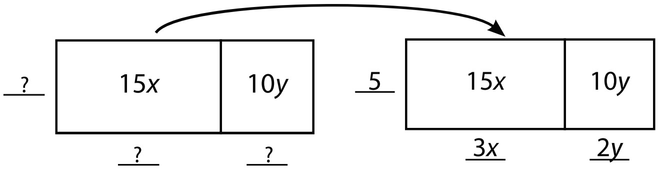
$P = 6x - 2$

If $x = 6$, then $P = 6(6) - 2 = 34$ cm

Area of a Rectangle

Example: The areas of two adjoining rectangles are given. Find the lengths and widths of the rectangles. Let the width of the rectangles be the GCF of the two areas. Find each length. Fill in the blanks.

1. The GCF of the two areas is 5.
2. Factor out the 5 from $15x$ to get a length of $3x$.
3. Factor 5 out of $10y$ to get a length of $2y$.



Order of Operations

When evaluating a numerical expression containing multiple operations, use a set of rules called the order of operations. The **order of operations** determines the order in which operations should be performed.

The order of operations is as follows:

Step ❶: Parentheses

Step ❷: Exponents

Step ❸: Multiplication/Division (left to right in the order that they occur)

Step ❹: Addition/Subtraction (left to right in the order that they occur)

If parentheses are enclosed within other parentheses, work from the inside out.

To remember the order, use the mnemonic device **"Please Excuse My Dear Aunt Sally."**

Use the following examples to help you understand how to use the order of operations.

Example: $2^2 + 6 \times 5$

To evaluate this expression, work through the steps using the order of operations.

Since there are no parentheses skip Step 1.

According to Step 2, simplify the exponents next.

Step 3 says to perform multiplication and division.

Next, Step 4 says to perform addition and subtraction.

$2^2 + 6 \times 5$ ⟵ Step ❷ Exponents

$4 + 6 \times 5$ ⟵ Step ❸ Multiplication and Division

$4 + 30$ ⟵ Step ❹ Addition and Subtraction

34

Example: $42 \div 6 - 3 + 4 - 16 \div 2$

Perform multiplication and division first (in the order they occur).

Perform addition and subtraction next (in the order they occur).

$42 \div 6 - 3 + 4 - 16 \div 2$ ⟵ Step ❸ Multiply and Divide

$7 - 3 + 4 - 16 \div 2$

$7 - 3 + 4 - 8$

$4 + 4 - 8$ ⟵ Step ❹ Add and Subtract

$8 - 8$

0

Help Pages

Expressions and Equations 7.EE.1 – 7.EE.4

Order of Operations (continued)

Example: $5(2 + 4) + 15 \div (9 - 6)$

Perform operations inside of parentheses first.
Perform multiplication and division first (in the order they occur).
Perform addition and subtraction next (in the order they occur).

$$\begin{aligned} 5(2+4)+15\div(9-6) &\longleftarrow \text{Step ❶Parentheses (Do operations inside first)} \\ 5(6)+15\div(3) &\longleftarrow \text{Step ❸Multiply and Divide (In the order they occur.)} \\ 30+5 &\longleftarrow \text{Step ❹Add} \\ 35 & \end{aligned}$$

Example: $4[3 + 2(7 + 5) - 7]$

Brackets are treated as parentheses. Start from the innermost parentheses first.
Then work inside the brackets.

$$\begin{aligned} 4[3+2(7+5)-7] &\longleftarrow \text{Step ❶Parentheses (including brackets) Solve innermost parentheses first. Then work inside the brackets.} \\ 4[3+2(12)-7] & \\ 4[3+24-7] & \\ 4[27-7] & \\ 4[20] & \\ 80 & \end{aligned}$$

Example: Place grouping symbols to make this equation true. $36 \div 4 + 5 = 4$

	Without Grouping Symbols	With Grouping Symbols	
Without parentheses the first step is 36÷4. ⟶	$36 \div 4 + 5 = ?$	$36 \div (4 + 5) = ?$	⟵ With parentheses the first step is 4 + 5.
	$9 + 5 = ?$	$36 \div 9 = ?$	
	$9 + 5 = 14$	$36 \div 9 = 4$	

Inequalities

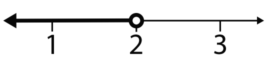
An inequality is a statement that one quantity is different than another (usually larger or smaller). The symbols showing inequality are <, >, ≤, and ≥ (less than, greater than, less than or equal to, and greater than or equal to). An inequality is formed by placing one of the inequality symbols between two expressions. The solution of an inequality is the set of numbers that can be substituted for the variable to make the statement true.

A simple inequality is $x \le 4$. The solution set, {..., 2, 3, 4}, includes all numbers that are either less than four or equal to four.

Inequalities can be graphed on a number line. For < and >, use an open circle; for ≤ and ≥, use a closed circle.

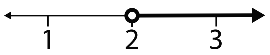
Example 1

$x < 2$



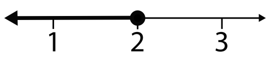
Example 2

$x > 2$



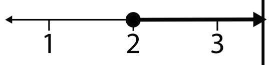
Example 3

$x \le 2$



Example 4

$x \ge 2$



Some inequalities are solved using only addition or subtraction. The approach to solving them is similar to that used when solving equations. The goal is to get the variable alone on one side of the inequality and the numbers on the other side.

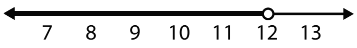
Examples: Solve $x - 4 < 8$

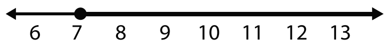
$$\begin{array}{lcr} x - 4 & < & 8 \\ +4 & & +4 \\ \hline x & < & 12 \end{array}$$

1. To get the variable alone, add the opposite of the number that is with it to both sides.
2. Simplify both sides of the inequality.
3. Graph the solution on a number line. For < and >, use an open circle; for ≤ and ≥, use a closed circle.

Solve $y + 3 \ge 10$

$$\begin{array}{lcr} y + 3 & \ge & 10 \\ -3 & & -3 \\ \hline y & \ge & 7 \end{array}$$





Help Pages

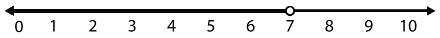
Expressions and Equations 7.EE.1 – 7.EE.4

Inequalities (continued)

Some inequalities are solved using only multiplication or division. The approach to solving them is also similar to that used when solving equations. Here, too, the goal is to get the variable alone on one side of the inequality and the numbers on the other side.

Example: Solve $8n < 56$.

$$\frac{8n}{8} < \frac{56}{8}$$
$$n < 7$$



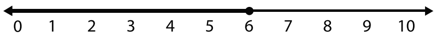
1. Check to see if the variable is being multiplied or divided by a number.
2. Use the same number, but do the opposite operation on both sides.
3. Simplify both sides of the inequality.
4. Graph the solution on a number line. For < and >, use an open circle; for ≤ and ≥, use a closed circle.

Some inequalities must be solved using both addition/subtraction and multiplication/division. In these problems, the addition/subtraction is always done first. (When multiplying or dividing by a negative, the inequality switches to its opposite.)

Example: Solve $2x - 6 \leq 6$.

$$\begin{array}{rcr} 2x - 6 & \leq & 6 \\ +6 & & +6 \\ \hline 2x & \leq & 12 \end{array}$$

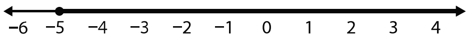
$$\frac{2x}{2} \leq \frac{12}{2}$$
$$x \leq 6$$



Example: Solve $-3n + 7 \leq 22$.

$$\begin{array}{rcr} -3n + 7 & \leq & 22 \\ -7 & & -7 \\ \hline -3n & \leq & 15 \end{array}$$

$$\frac{-3n}{-3} \geq \frac{15}{-3}$$
$$n \geq -5$$

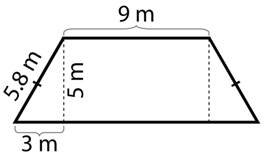


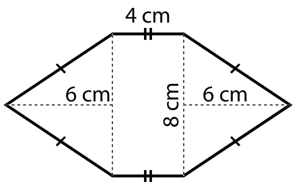
Geometry 7.G.1 – 7.G.6

Congruent Sides and Angles

One way to denote **congruent sides** is to use small line segments called hash marks. The number of marks indicates which line segments are congruent.

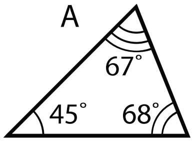
For example, the left and right sides of the trapezoid each measure 5.8 m, and a single hash mark on each shows that they are congruent.

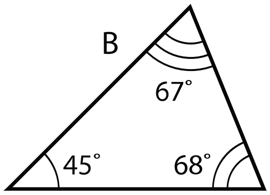




The irregular hexagon has six sides; four of the sides have single hash marks, indicating that all four are congruent and have the same slant height. Two sides each measure 4 cm, so the double hash marks indicate that they are congruent.

Similarly, corresponding congruent angles are designated with arcs. The number of arcs indicate which angles are congruent. Even though triangle B is larger than triangle A, the two are similar. The single arc denotes congruent angles on both triangles (45°). A double arc denotes two other congruent angles (68°), and a triple arc denotes another set of congruent angles (67°).



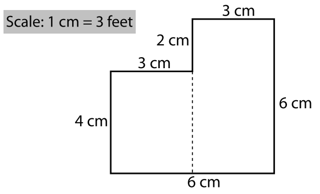


Help Pages

Geometry　　7.G.1 – 7.G.6

Using Scale to Find Perimeter and Area

The drawing to the right is a model representing a room. According to the key, each centimeter in this drawing is equal to 3 feet in the actual room. This relationship, 1 cm = 3 ft, is called the **scale**. The **scale factor** is the ratio that relates the size of the two objects.



Example: Use the scale to find the perimeter and area of the room represented by the model.

Perimeter:
1. Multiply the scale factor by each measure given in the model to find the actual length of each wall.
2. Use the actual lengths to calculate perimeter.

Area:

3. Partition the shape into two rectangles.
4. Then use the actual lengths again to calculate area.

$4\text{ cm}\times\frac{3\text{ ft}}{1\text{ cm}}=12\text{ ft}$

$3\text{ cm}\times\frac{3\text{ ft}}{1\text{ cm}}=9\text{ ft}$

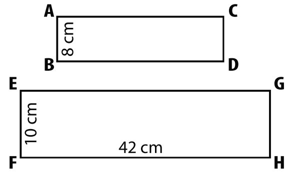
$2\text{ cm}\times\frac{3\text{ ft}}{1\text{ cm}}=6\text{ ft}$

$6\text{ cm}\times\frac{3\text{ ft}}{1\text{ cm}}=18\text{ ft}$

Perimeter:
$12+9+6+9+18+18$
$=72$ ft

Area:
$(12\times9)+(9\times18)$
$=108+162=270\text{ ft}^2$

Example: Given two similar shapes, you can determine the scale by comparing side lengths. Compare the length of side AB to the length of side EF. Then, write a ratio to represent the scale.



1. Write a fraction representing the ratio of $\overline{EF}$ to $\overline{AB}$.
2. Reduce the fraction to find the scale.

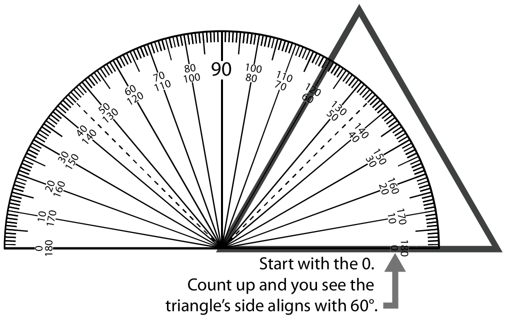
$\frac{10\text{ cm}}{8\text{ cm}}=\frac{5\text{ cm}}{4\text{ cm}}$

Scale: 5 cm : 4 cm

Using a Protractor

A protractor can be used to measure an angle. The picture at right shows a protractor over a triangle.

Example: Measure the angle of the triangle.

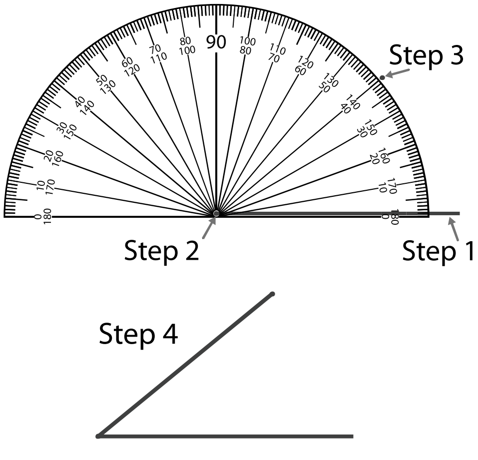


Start with the 0.
Count up and you see the triangle's side aligns with 60°.

1. The bottom edge of this protractor lines up with that of the triangle. (Protractors may differ.)
2. The center of the edge (where all the lines converge) is placed directly over the triangle's vertex.
3. You can see that the left side of the triangle lines up exactly with the number 60. This means the measure of the angle is 60°.

A protractor can also be used to draw an angle.

Example: Draw a 40° angle.



1. First, use the bottom edge of your protractor to draw a straight line. This will form one of the rays of your angle.
2. Your protractor will probably have a hole at the point where all lines intersect. Place this hole over the endpoint of your line and make a dot. This will be your vertex.
3. Now, place a dot at the end of the 40°. This will be the endpoint of your second ray.
4. Finally, remove the protractor and connect the two dots. You now have a forty degree angle. If you need the lines to be a different length when drawing a triangle, you can place a new dot at any point on the line and erase the rest. The angle will remain the same.

Help Pages

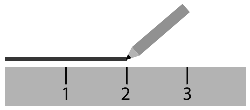
Geometry 7.G.1 – 7.G.6

Drawing a Triangle from the Side Lengths

Example: Draw a triangle with side lengths of 2, 3, and 4 in.

1. Using a ruler, draw a 2 inch line.
2. Label the endpoints A and B.
3. Using the ruler, stretch your compass to a width of 3 in.
4. Place the point of the compass on point A of your line and draw an arc.
5. Stretch your compass to 4 in. Place the point of the compass on point B of your line and draw an arc. Point C is the point where the arcs intersect.
6. Connect the lines. You now have a triangle ABC with side lengths 2, 3, and 4 in.

❶



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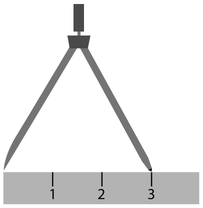
❸



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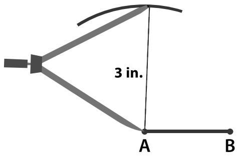

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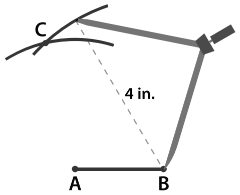

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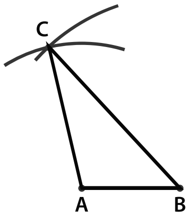
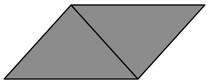


Finding the Area of a Triangle

To find the area of a triangle, it is helpful to recognize that any triangle is exactly half of a paralleogram.

The whole figure is a parallelogram.

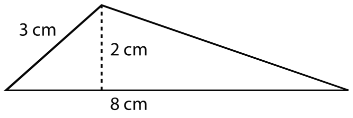


Half of the whole figure is a triangle.

So, the triangle's area is equal to half of the product of the base and the height.

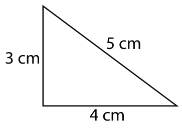
$$\textbf{Area of triangle} = \tfrac{1}{2}(\textbf{\textit{base}} \times \textbf{\textit{height}}) \textbf{ or } \textbf{\textit{A}} = \tfrac{1}{2}\textbf{\textit{bh}}$$

Examples: Find the area of the triangles below.



$A = 8\text{ cm} \times 2\text{ cm} \times \frac{1}{2} = 8\text{ cm}^2$

1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down — never slanted.)
3. Multiply them together and divide by 2 to find the area. (8 cm^2)



The base of this triangle is 4 cm long.

Its height is 3 cm. (Remember, the height is always straight up and down!)

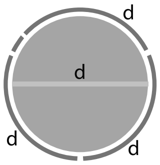
$A = 4\text{ cm} \times 3\text{ cm} \times \frac{1}{2} = 6\text{ cm}^2$

Help Pages

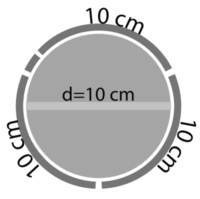
Geometry 7.G.1 – 7.G.6

What is pi?

For any circle, π is the ratio of the diameter to the circumference. The value of **pi** is approximately 3.14 or $\frac{22}{7}$. In other words, the circumference of a circle is equal to a little more than 3 times the diameter. From this we get the formula for the circumference of a circle: $C = \pi d$ or $C = 2\pi r$.

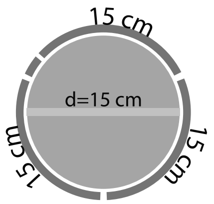


Example 1



$C = \pi d = 31.4$

Example 2



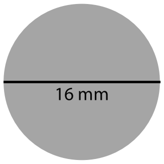
$C = \pi d = 47.1$

Circumference of a Circle

The **circumference** of a circle is the distance around the outside of the circle. Before you can find the circumference of a circle you must know either its radius or its diameter. Once you have this information, the circumference can be found by multiplying the diameter by pi (π).

Circumference = C = π × *diameter*

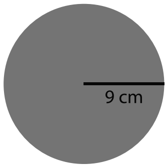
Example: Use the diameter to find the circumference of the circle below.



1. Find the length of the diameter. (16 mm)
2. Multiply the diameter by π. (16 mm × 3.14)
3. The product is the circumference.
   16 mm × 3.14 = 50.24 mm

Sometimes the radius of a circle is given instead of the diameter. Remember, the radius of any circle is exactly half of the diameter. If a circle has a radius of 3 feet, its diameter is 6 feet.

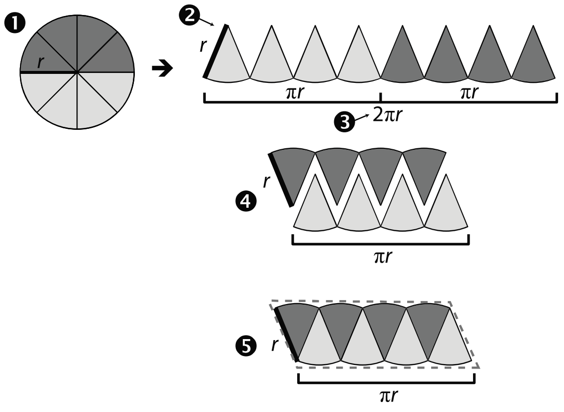
Example: Use the radius to find the circumference of the circle below.



1. Since the radius is 9 cm, the diameter must by 18 cm.
2. Multiply the diameter by π. (18 cm × 3.14)
3. The product is the circumference.
   18 cm × 3.14 = 56.52 cm

Finding the Area of a Circle

The area of a circle is related to its circumference. One way to visualize the relationship between circumference and the area of a circle is to compare the circle to a parallelogram.



1. Consider the circle to be a series of wedges. If you "unroll" the wedges it forms a row.
2. The height of the row is equal to the radius of the circle (r).
3. Note that the length of the row is equal to the circumference of the circle ($2\pi r$).
4. Now fold over the row of wedges. You have a row that is half the distance of your circumference (πr). The height of the row is still equal to r.
5. Note that the shape formed is similar to a parallelogram.
6. The formula for the area of a parallelogram is $b \times h$. Here the base is πr and the height is r. So the area of this shape is approximately πr^2.
7. The formula for the area of a circle is πr^2.

Help Pages

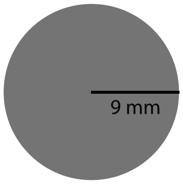
Geometry 7.G.1 – 7.G.6

Finding the Area of a Circle (continued)

When finding the area of a circle, the length of the radius is squared (multiplied by itself), and then that answer is multiplied by the constant, pi π. $\pi = 3.14$ (rounded to the nearest hundredth) or $\frac{22}{7}$.

$$\textbf{Area} = \pi \times \textbf{radius} \times \textbf{radius} \quad \textbf{or} \quad \textbf{A} = \pi r^2$$

Example: Use the radius to find the area of the circle below.

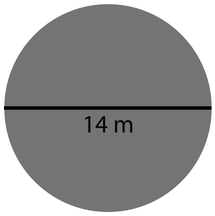


1. Find the length of the radius. (9 mm)
2. Multiply the radius by itself. (9 mm × 9 mm)
3. Multiply the product by pi. ($81\text{ mm}^2 \times 3.14$)
4. The result is the area. (254.34 mm^2)

$$A = 9\text{ mm} \times 9\text{ mm} \times 3.14 = 254.34\text{ mm}^2$$

Sometimes the diameter of a circle is given instead of the radius. Remember, the diameter of any circle is exactly twice the radius. If a circle has a diameter of 6 feet, its radius is 3 feet.

Example: Use the diameter to find the area of the circle below.



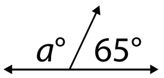
1. Since the diameter is 14 m, the radius must be 7 m.
2. Square the radius. (7 m × 7 m)
3. Multiply that result by π. ($49\text{ m}^2 \times \frac{22}{7}$)
4. The product is the area. (154 m^2)

$$A = 49\text{ m}^2 \times \frac{22}{7} = 154\text{ m}^2$$

Types of Angles

Supplementary angles are two angles whose sum is 180°.

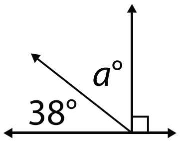
Example: The graphic shows an example of supplementary angles. What is the measure of angle *a*?



1. Because the angles are supplementary, the sum of the two angles is 180°.
2. $180 - 65 = 115$
3. The measure of angle *a* is 115°.

Complementary angles are two angles whose sum is 90°.

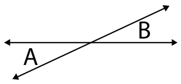
Example: The image shows an example of complementary angles. Give the value of angle *a*.



1. Because the angles are complementary, the sum of the two angles is 90°.
2. $90 - 38 = 52$
3. The measure of angle *a* is 52°.

Vertical angles are created when two lines intersect. Vertical angles are always congruent.

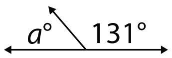
Example: Angles *A* and *B* are vertical angles. If the measure of angle *A* is 25°, what is the measure of angle *B*?



1. Because the angles are vertical, the two angles are congruent.
2. If angle *A* = 25°, then angle *B* = 25° also.

Adjacent angles share a common vertex and a common side, but do not overlap. They are created when two lines intersect; they are directly next to one another.

Example: The adjacent angles shown here are supplementary. Give the value of *a*.



1. Because the angles are supplementary, the sum of the two angles is 180°.
2. $180 - 131 = 49$
3. The measure of angle *a* is 49°.

Help Pages

Geometry 7.G.1 – 7.G.6

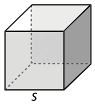
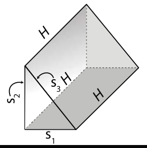
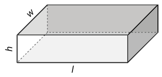
Finding the Area of a Regular Polygon

circle	$A = \pi r^2$	square	$A = s^2$
parallelogram	$A = b \times h$	trapezoid	$A = \frac{(b_1 + b_2) \times h}{2}$ or $\frac{1}{2}(b_1 + b_2) \times h$
rectangle	$A = l \times w$	triangle	$A = \frac{1}{2}(b \times h)$

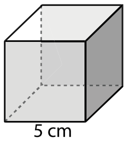
Solid Figures – Prisms

Definition – a three-dimensional figure that has two identical, parallel bases and three or more rectangular faces. (There are as many faces as there are sides on the bases.) A prism gets its name from the shape of its bases.

Types of Prisms

Figure	Description	Example	Volume	Surface Area
cube	square bases and faces; all edges, faces, and vertices are congruent (blocks, number cubes, etc.)		s^3 or $l \times w \times h$	$6s^2$ where s is the side length
triangular prism	a prism with two triangular bases and three rectangular faces (type of prism that bends light and creates a rainbow)		Bh area of triangular base × height (h) of prism	(areas of two bases) + (areas of 3 faces)
right rectangular prism	a prism with rectangular bases and four rectangular faces (shoe box, refrigerator, pizza box)		$l \times w \times h$	$2(lw + lh + hw)$

Example: Calculate the volume and surface area of this cube.



To Calculate Volume:

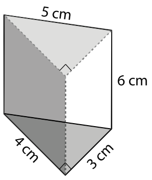
1. Because this is a cube, all sides have the same length. In this cube, the side length is 5 cm.
2. For a cube volume (V) = s^3, so V = 5^3 = 125 cm^3

To Calculate Surface Area

1. For a cube, surface area = $6s^2$, so S.A. = $6(5^2)$ = 150 cm^2

Volume:
$V = 5^3 = 125\ cm^3$
Surface Area:
$S.A. = 6s^2 = 6(5^2) = 150\ cm^2$

Example: Calculate the volume and surface area of this triangular prism.



To Calculate Volume:

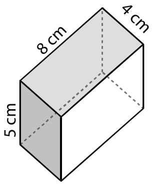
1. First, find the area of the triangular base ($A = \frac{1}{2}bh$).
2. Multiply the area by the height of the triangle.

To Calculate Surface Area

1. Find the area for the two triangular bases.
2. Because there are two bases, double this number.
3. Find the area for each rectangular side. All three rectangles have a height of 6 cm. Multiply 6 times each side length.
4. Add the area of the two bases to the area of each rectangle.

Volume:
$A = \frac{1}{2}(3 \times 4) = 6\ cm^2$
$V = 6 \times 6 = 36\ cm^3$
Surface Area:
❶ $A = \frac{1}{2}bh = \frac{1}{2}(3 \times 4) = 6\ cm^2$
❷ $6 \times 2 = 12\ cm^2$
❸ $\begin{cases} 6 \times 3 = 18\ cm^2 \\ 6 \times 5 = 30\ cm^2 \\ 6 \times 4 = 24\ cm^2 \end{cases}$
❹ $12 + 18 + 30 + 24 = 84\ cm^2$

Example: Calculate the volume and surface area of this right rectangular prism.



To Calculate Volume:

1. The formula for volume of a rectangular prism is $l \times w \times h$.

To Calculate Surface Area

1. Find the area of three different sides.
2. Add the areas and then double.

Volume:
$5 \times 8 \times 4 = 160\ cm^3$
Surface Area:
❶ $4 \times 5 = 20\ cm^2$
$8 \times 4 = 32\ cm^2$
$8 \times 5 = 40\ cm^2$
❷ $20 + 32 + 40 = 92$
❸ $92 \times 2 = 184\ cm^2$

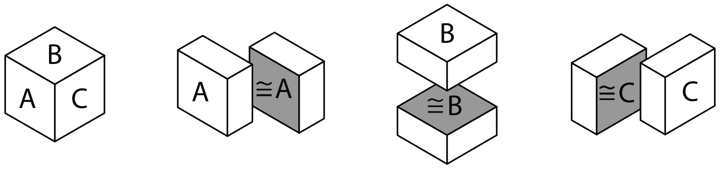
Help Pages

Geometry 7.G.1 – 7.G.6

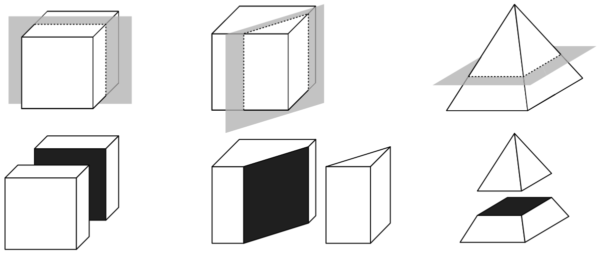
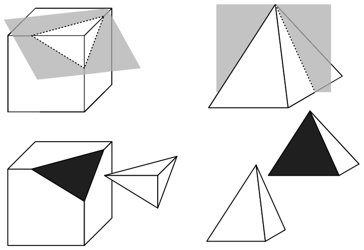
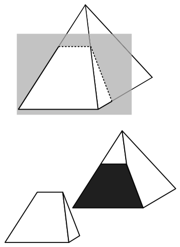
Solid Figures – Cross sections

A **cross section** is a face formed when you slice through a 3D object.

Imagine taking a cross section of a rectangular prism. In this example, if the slice is parallel to a face, the resulting cross section will be a rectangle **congruent** (≅) to that face. Each picture shows the same rectangle sliced by a plane.



In the examples below, observe the different faces that result from each cross section.

Rectangular Cross-sections	Triangular Cross-sections	Trapezoidal Cross-sections
		

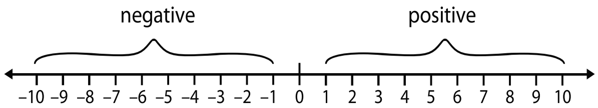
Help Pages

The Number System

7.NS.1 – 7.NS.3

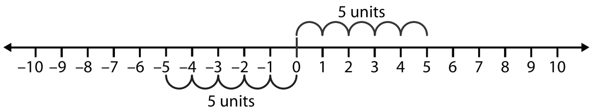
Positive and Negative Numbers

Integers include the counting numbers, their opposites (negative numbers), and zero.



When **ordering integers**, arrange them either from least to greatest or from greatest to least. The further a number is to the right on the number line, the greater its value. For example, −2 is further to the right than −6, so −2 is greater than −6.

The **absolute value of a number** is its distance from zero on a number line. It is never negative.



The absolute value of both −5 and +5 is 5, because both are 5 units away from zero. The symbol for the absolute value of −5 is |−5|. Examples: |−3| = 3; |8| = 8

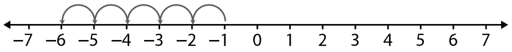
Addition

Operation	Addend	Addend	Sum
Addition	+	+	+
	−	−	−
	+	−	Subtract the integers, then take the sign of the integer with the greater absolute value.
	−	+	

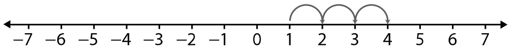
Example: Use the number line to find the sums. Note the signs of the addends. What do you notice about the sign of each sum?

−1 + −5 = ?   1 + 3 = ?

−1 + −5 = −6



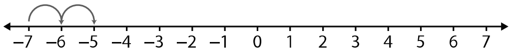
1 + 3 = 4



The sum is positive when both addends are positive and the sum is negative when both addends are negative.

Example: Find the sum of −7 + 2. Note the different signs of the addends.

−7 + 2 = −5



The sum is negative because the negative addend has a greater absoute value than the positive addend. The movement in the positive direction does not go past zero.

Subtraction

When working with integers, rewrite subtraction problems as addition problems. Subtracting an integer is the same as adding its opposite.

Example: 5 – 3 is the same as 5 + (–3). The solution to both problems is 2. Use that process for subtracting all integers.

Here are more examples:

8 – 3 = 8 + (–3) = +5   8 – (–3) = 8 + (+3) = +11   –8 – 3 = –8 + (–3) = –11   –8 – –3 = –8 + (+3) = –5

Help Pages

The Number System 7.NS.1 – 7.NS.3

Multiplication and Division

Operation	Factor (Dividend)	Factor (Divisor)	Product (Quotient)
Multiplication (or Division)	+	+	+
	−	−	+
	+	−	−
	−	+	−

Example: Solve. What is the sign of the product?

-2×-3 When two factors have the same sign, the product is positive. $-2 \times -3 = 6$

-2×4 When two factors have different signs, the product is negative. $-2 \times 4 = -8$

Fractions and Decimals

Any fraction can be expressed as a decimal. Divide the numerator by the denominator.

Example: Use long division to write $\frac{2}{5}$ as a decimal.

$$\begin{array}{r} 0.4 \\ 5\overline{)2.000} \\ \underline{-2.0} \\ 0 \end{array}$$

$\frac{2}{5} = 0.4$

When a fraction is converted to a decimal, sometimes there is a repeating decimal. Show a repeating decimal by drawing a bar over the digits that repeat. The fraction $\frac{1}{3}$ converts to the decimal 0.333... We can write this as $0.\overline{3}$.

Example: Use long division to write $\frac{1}{6}$ as a decimal with the proper notation.

$$\begin{array}{r} 0.1666 \\ 6\overline{)1.0000} \\ \underline{-0.6} \\ 40 \\ \underline{-36} \\ 40 \\ \underline{-36} \\ 40 \end{array}$$

$\frac{1}{6} = 0.1\overline{6}$

Fractions – Undefined

In a fraction, the denominator cannot be zero. Because of the inverse relationship, you know that $\frac{0}{4}$ or $0 \div 4 = 0$. This does not work with $\frac{4}{0}$ or $4 \div 0$ because $0 \times 0 \neq 4$. **If zero is in the denominator of a fraction, the answer is undefined.**

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Calculating Percents and Proportions

Percent literally means "per hundred." For example, to find 50% of a number, multiply the number by $\frac{50}{100}$.

A **proportion** is a statement that two ratios are equal to each other. There are two ways to solve a proportion when a number is missing.

One way to solve a proportion is by using cross-products.

Example: $\frac{14}{20} = \frac{21}{n}$. Solve for n.

1. Multiply downward on each diagonal.
2. Make the product of each diagonal equal to each other.
3. Solve for the missing variable.

$$\frac{14}{20} = \frac{21}{n}$$

$$\frac{14}{20} \times \frac{21}{n}$$

$$14n = 420$$

$$\frac{14n}{14} = \frac{420}{14}$$

$$n = 30$$

Another way to solve a proportion is already familiar to you. You can use the equivalent fraction method.

Example: $\frac{5}{8} = \frac{n}{64}$. Solve for n.

1. To make $\frac{n}{64}$ equivalent to $\frac{5}{8}$, multiply both numerator and denominator by 8. Remember $\frac{8}{8}$ is another name for 1.

$\frac{5}{8} \times \frac{8}{8} = \frac{40}{64}$, $n = 40$

Help Pages

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Calculating Percents and Proportions (continued)

The equivalent fraction method can be used with percent problems, as well.

Example: 40 of 50 sixth graders went on a field trip. What percent of the sixth graders went on the trip?

1. Write the given information as a fraction.
2. Set up the proportion. Percent literally means "per hundred" so the denominator for the second fraction will be 100.
3. The ratio represents part to whole. 40 out of 50 sixth graders is equal to the unknown percent to 100 percent (or the whole).
4. $\frac{40}{50} \times \frac{2}{2} = \frac{80}{100}$; $\frac{80}{100}$ means 80 per 100 or 80%

40 of 50 is $\frac{40}{50}$

$\frac{40}{50} = \frac{n}{100}$

$\frac{40}{50} = \frac{\boxed{?}}{100}$

$\frac{40}{50} = \frac{\boxed{?}}{100} = \underline{\quad}\%$

When changing between fraction, decimals and percents, it is very helpful to use an FDP chart (Fraction, Decimal, Percent).

To **change a fraction to a percent and/or decimal**, first find an equivalent fraction with 100 in the denominator. Once you have found that equivalent fraction, it can easily be writen as a decimal. To change that decimal to a percent, multiply by 100 (move the decimal point 2 places to the right) and add a % sign.

Example: Change $\frac{2}{5}$ to a percent and then to a decimal.

When **changing from a percent to a decimal or a fraction**, the process is similar to the one used above. Begin with the percent. Write it as a fraction with a denominator of 100; reduce this fraction. Return to the percent, drop the percent symbol, divide by 100 (move the decimal point 2 places to the left.) This is the decimal.

1. Find an equivalent fraction with 100 in the denominator.
2. From the equivalent fraction above, the decimal can easily be found. Say the name of the fraction: "forty hundredths." Write this as a decimal: 0.40
3. To change 0.40 to a percent, multiply by 100. Add a % sign.

❶ $\underset{\times 20}{\overset{\times 20}{\frac{2}{5} = \frac{?}{100}}}$ $? = 40$

❷ $\frac{2}{5} = \frac{40}{100} = 0.40$

❸ $0.40 = 40\%$

F	D	P
$\frac{2}{5}$		

F	D	P
$\frac{2}{5} = \frac{?}{100}$	0.40	

F	D	P
$\frac{2}{5} = \frac{40}{100}$	0.40	40%

Example: Write 45% as a fraction and then as a decimal.

1. Begin with the percent. Write a fraction where the denominator is 100 and the numerator is the "percent."
2. This fraction must be reduced.
3. Go back to the percent. Drop the % symbol. Divide by 100 to change it to a decimal.

❶ $45\% = \frac{45}{100}$

❷ $\frac{45(\div 5)}{100(\div 5)} = \frac{9}{20}$

❸ $45\% = .45$ Decimal point goes here.

When changing from a decimal to a percent or fraction, again, the process is similar to the one used above. Begin with the decimal. Multiply by 100 and add a percent sign (%). Return to the decimal. Write it as a fraction and reduce.

Example: Write 0.12 as a percent and then as a fraction.

1. Begin with the decimal. Multiply by 100 to change it to a percent.
2. Go back to the decimal and write it as a fraction. Reduce this fraction.

❶ $0.12 = 12\%$

❷ $0.12 =$ twelve hundredths

❸ $= \frac{12}{100} = \frac{12(\div 4)}{100(\div 4)} = \frac{3}{25}$

Help Pages

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Unit Rates and Complex Fractions

Complex fractions are fractions that have fractions in the numerator and/or fractions in the denominator.

Example: Which of the following are complex fractions? A) $\frac{\frac{4}{5}}{6}$ B) $\frac{4}{5}$ C) $\frac{\frac{2}{3}}{\frac{6}{8}}$ D) $8\frac{1}{3}$

A) Yes. There is a fraction in the numerator.
B) This is not a complex fraction.
C) Yes. There are fractions in both the numerator and denominator.
D) This is not a complex fraction; it is a mixed number.

A complex fraction can be simplified. One way to simplify a complex fraction would be as a proportion where the new fraction would have a denominator of one. (Remember, we call this the unit rate.)

Example: Frank bikes $\frac{9}{10}$ mile in $\frac{1}{12}$ of an hour. Find Frank's average speed in miles per hour.

$$\frac{\overset{\times\frac{12}{1}}{\frac{9}{10}}}{\underset{\times\frac{12}{1}}{\frac{1}{12}}} = \frac{?}{1}$$

$$\frac{\overset{\times\frac{12}{1}}{\frac{9}{10}}}{\underset{\times\frac{12}{1}}{\frac{1}{12}}} = \frac{\frac{108}{10}}{\frac{12}{12}} = \frac{\frac{108}{10}}{1} = \frac{108}{10} = 10.8$$

1. Construct a proportion in which the unknown fraction has a denominator of 1.
2. Determine what factor times the denominator will equal 1. In this example, a factor of $12 \times \frac{1}{12} = 1$.
3. Multiply both the numerator and the denominator times this factor.
4. Simplify.

Another way to approach simplifying a complex fraction is to treat it as a division problem.

Example: $\frac{\frac{2}{3}}{5}$ is a complex fraction. Simplify.

$$\frac{\frac{2}{3}}{5}$$

$$\frac{2}{3} \div 5 = ?$$

$$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$$

1. Rewrite the complex fraction as a division problem.
2. Multiply by the reciprocal to solve.

Example: Simply the complex fraction $\frac{\frac{1}{4}}{\frac{6}{8}}$.

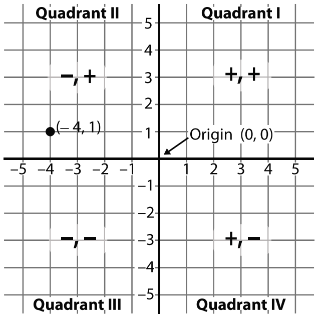
$$\frac{\frac{1}{4}}{\frac{6}{8}} \text{ is } \frac{1}{4} \div \frac{6}{8}$$

$$\frac{1}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{6} = \frac{2}{6} = \frac{1}{3}$$

Help Pages

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Graphing on a Coordinate Plane

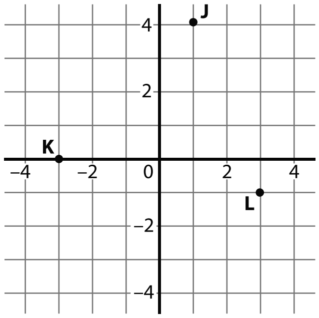


A **coordinate plane** is formed by the intersection of a horizontal number line, called the ***x*-axis**, and a vertical number line, called the ***y*-axis.** The axes meet at the point (0, 0), called the **origin**, and divide the coordinate plane into four **quadrants**.

Points are represented by ordered pairs of numbers, (*x*, *y*). The first number in an **ordered pair** is the *x*-coordinate; the second number is the *y* coordinate. In the point **(–4, 1)**, –4 is the *x*-coordinate and 1 is the *y*-coordinate.

When graphing on a coordinate plane, always move on the *x*-axis first (right or left), and then move on the *y*-axis (up or down).

- The coordinates of point J are (1, 4).
- The coordinates of point K are (–3, 0).
- The coordinates of point L are (3, –1).



Proportional Relationships

If quantities are in a **proportional relationship**, the ratios are equivalent to each other.
Example: Is the relationship shown in this table proportional?

Gallons (*x*)	Cost of Gas (*y*)
2	8
5	20
7	28
8	32

1. To determine if the quantities are proportional, find the unit rate for each data set. What is the cost for 1 gallon of gas?
2. Compare the ratios for each data set. In this table, the ratios are all equal.

$\frac{8}{2} = \$4$ per gallon

$\frac{20}{5} = \$4$ per gallon

$\frac{28}{7} = \$4$ per gallon

$\frac{32}{8} = \$4$ per gallon

$\frac{8}{2} = \frac{20}{5} = \frac{28}{7} = \frac{32}{8} = 4$

This relationship is proportional.

Example: Is the relationship shown in this table proportional?

Numbers of Students	Pieces of Pizza
2	6
3	15
7	21
8	24

1. Find the unit rate. How many slices of pizza per student?
2. Compare the ratios for each data set. In this table, the ratios are not all equal.

$\frac{6}{2} = 3$ pieces per student

$\frac{15}{3} = 5$ pieces per student

$\frac{21}{7} = 3$ pieces per student

$\frac{24}{8} = 3$ pieces per student

$\frac{6}{2} = \frac{21}{7} = \frac{24}{8} = 3; \frac{15}{3} \neq 3$

This relationship is <u>not</u> proportional.

Help Pages

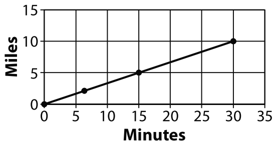
Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Proportional Relationships (continued)

Graphs and Proportional Relationships

When the data from a proportional relationship is graphed on a coordinate plane, all of the points lie on a straight line. In addition, the line always passes through the origin (0, 0).

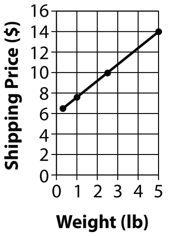
Example: Does this graph show a proportional relationship?



1. The points in the relationship **do** form a straight line.
2. The line **does** pass through the origin.

Therefore, this graph **does** represent a proportional relationship.

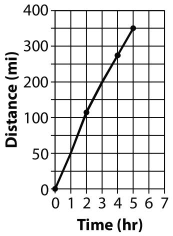
Example: Does this graph show a proportional relationship?



1. The points in the relationship **do** form a straight line.
2. The line **does not** pass through the origin.

Therefore, this graph **does not** represent a proportional relationship.

Example: Does this graph show a proportional relationship?



1. The points in the relationship **do not** form a straight line.
2. The line **does** pass through the origin.

Therefore, this graph **does not** represent a proportional relationship.

Percent – Markdown and Markup Problems

One type of percent problem would be a **markdown**. A sale is an example of a markdown.
Example: Jamie wants to buy a $60 blouse. The store is having a 15% off sale. What will Jamie pay for the blouse?

There are two ways to solve this problem.

Method 1:
1. Determine 15% of $60.
2. A markdown involves subtraction. Subtract the markdown amount from the original cost of $60.
3. The difference is the marked down price.

$$15\% = 0.15$$
$$x = 0.15(\$60)$$
$$x = 9$$
$$\$60 - 9 = \$51$$

Method 2:
1. Subtract 15% from 100% to find the percent Jamie will be charged.
2. Multiply this percent times the original cost.
3. The product will be the new cost.

$$100 - 15 = 85\%$$
$$85\% = 0.85$$
$$x = 0.85(\$60)$$
$$x = \$51$$

Help Pages

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Percent – Markdown and Markup Problems (continued)

Another type of a percent problem would be a **markup**. A store buys merchandise from a manufacturer. They then mark up the cost of the merchandise in order to make a profit when they sell it to the customer.

Example: The clothing at The Chic Boutique marks up its merchandise by 23%. The boutique bought a sweater from the manufacturer for $95. What price will they sell the sweater for?

Method 1:
1. Determine 23% of $95.
2. A markup is an added cost. Add that amount to the original cost of $95.
3. The sum is the final cost.

$$23\% = 0.23$$
$$x = 0.23(\$95)$$
$$x = 21.85$$
$$\$95 + 21.85 = \$116.85$$

Method 2:
1. Add 23% to100%.
2. Multiply this percent times the original cost.
3. The product will be the new cost.

$$100 + 23 = 123\%$$
$$123\% = 1.23$$
$$x = 1.23(\$95)$$
$$x = \$116.85$$

Percent – Tax

Tax is another type of percent problem. A tax is a quantity added to the final price of an item.

Example: Susan wants to buy a $45 blouse. The sales tax is 6%. What will Jamie pay for the blouse?

Method 1:
1. Determine 6% of $45.
2. A tax is an added cost. Add that amount to the original cost of $45.
3. The sum is the final cost.

$$6\% = 0.06$$
$$x = 0.06(\$45)$$
$$x = 2.70$$
$$\$45 + 2.70 = \$47.70$$

Method 2:
1. Add 6% to100%.
2. Multiply this percent times the original cost.
3. The product will be the new cost.

$$100 + 6 = 106\%$$
$$106\% = 1.06$$
$$x = 1.06(\$45)$$
$$x = \$47.70$$

Example: Shanice wants to buy a $115 coat. The store is having a 15% off sale. The sales tax is 6%. What will Shanice pay for the coat?
This is both a markdown and a tax problem. Because tax is calculated based on the final cost, we will determine the markdown cost first.

Method 1:
1. Determine 15% of $115.
2. A markdown involves subtraction. Subtract the markdown amount from the original cost of $115.
3. The difference ($97.75) is the cost before tax.
4. Determine 6% of $97.75.
5. Remember, tax is an added cost. Add this amount to the cost.
6. The final amount is how much Shanice will pay.

$$15\% = 0.15$$
$$\text{markdown} = 0.15(\$115)$$
$$= \$17.25$$
$$\text{cost} = \$115 - 17.25 = \$97.75$$
$$6\% = 0.06$$
$$\text{tax} = 0.06(\$97.75)$$
$$= 5.87$$
$$\text{amount paid} = \$97.75 + 5.87 = \$103.62$$

Help Pages

Ratios & Proportional Relationships 7.RP.1 – 7.RP.3

Percent – Simple Interest

Interest is an *amount that is paid* when money is borrowed or an *amount that is earned* when money is loaned. The amount that is borrowed or loaned is called the principal. The principal plus the interest is paid by the borrower to the lender after an agreed upon time.

Simple interest is a percentage of the principal, and it adds up over the time of the loan. Simple interest is calculated using the formula:

principal × **rate** × **time** = **interest** or **prt** = **i**

For example, if you borrow $500 from a bank at 4% annual interest, $500 is the **principal**, 4% is the **rate**, and one year is the **time**. After one year, you would pay $520. Here is why:

$$\underbrace{(500 \times 0.04) \times 1}_{\text{simple interest}} = 500 + 20 = \$520$$

Example:

Juanita deposits $5,500 in a high-interest CD. The account earns 9% annual interest. If she doesn't add or subtract any money from the account for 3 years, how much will be in the account at the end of that time?

1. First identify the principal, the rate and the time.
2. Next, find the interest using the formula *prt* = *i*.
3. Finally, add the interest to the original investment.

Principal = $5,500
Rate = 9%
Time = 3 years

Interest 5,500 × .09 × 3 = $1,485
Total Value $5,500 + 1,485 = $6,985

Percent Error

Percent error is a measure of how inaccurate a measured value is by comparing the measured value to the accepted (or true) value.
Note: This is the same formula used to calculate a percent increase or decrease.

$$\left|\frac{\text{accepted value - measured value}}{\text{accepted value}}\right| \times 100 = \%\text{ error}$$

Example: Sean measured the amount of liquid in a graduated cylinder. He recorded a value of 33.4 ml. The accepted (true) volume was 32.5 ml. Calculate the percent error in Sean's measurement. (Note the absolute value sign.)

$$\left|\frac{32.5\ -\ 33.4}{32.5}\right| \times 100 = 2.76\%$$

Statistics and Probability 7.SP.1 – 7.SP.8

Populations and Samples

By studying a sample, we can learn about a population. A **population** is an entire group that you want to know information about, for example, all the students of a high school or all the citizens of a country. A smaller group within a population is a **sample**.

Example: Cardwick Candy Company is introducing a new candy bar. Their research department wants to know what flavor of candy school-age children would prefer. Because it is not possible to ask every child, they want to pick a sample of the population. Study each phrase; write **P** if it names the population or **S** if it names a sample.

______ Mrs. Smith's fifth grade class

______ Children between the ages of 7 and 12

Answer: __S__ Mrs. Smith's fifth grade class

__P__ Children between the ages of 7 and 12

A representative sample accurately reflects a population and allows a study of the population to be valid. For example, if we want to find out whether voters in California will support a particular candidate for governor, our survey sample would include Californians who are able to vote. Our sample should not include people from other states or people who are not able to vote.

Example: If we wanted to determine the winner of the next presidential election, which population should we study?

A) all American middle school students
B) all members of the United States Congress
C) all American voters

- Answer A is not a good choice because middle school students can't vote.
- Answer B is not a random sample.

Answer: The answer is C.

Help Pages

Statistics and Probability 7.SP.1 – 7.SP.8

Populations and Samples (continued)

An **unbiased sample** is fair; it accurately represents the total population being studied. One way to have an unbiased sample is by taking a random sample. **A random sample is a type of representative sample**. In a random sample, any member of the population is equally likely to be chosen as a representative of the population.

Example: Researchers want to find out whether a town's citizens favor replacing the town's library with a new library. Which group would provide the most representative sample?

A) a random sample of citizens of the town
B) the town's working librarians and retired librarians
C) residents who live closest to the current library

Choices B and C are probably biased. A sample needs to be random to be unbiased.

Answer: A

Finding the Mean Absolute Deviation

The **mean absolute deviation (MAD)** is the mean (average) of the differences between each data point and the mean of all the data points. To find the mean absolute deviation of a set of data, follow these steps.

Use this data set: 3 4 4 4 5 6 6 8

1. Find the mean of the data set. Add all the numbers and divide by the number of data points.
2. List each data point and its deviation (absolute value of its difference from the mean). For example, the mean is 5, so the data point 8 has a deviation of 3 because 8 is 3 points from 5.
3. Add the deviations, or differences.
4. Divide by the number of data points. This is the average.

❶ 3 + 4 + 4 + 4 + 5 + 6 + 6 + 8 = 40 40 ÷ 8 = 5 Mean = 5

❷

Data Point	3	4	4	4	5	6	6	8
Deviation	2	1	1	1	0	1	1	3

❸ 2 + 1 + 1 + 1 + 0 + 1 + 1 + 3 = 10

❹ 10 ÷ 8 = 1.25 MAD = 1.25

Interpreting Data – Dot Plots

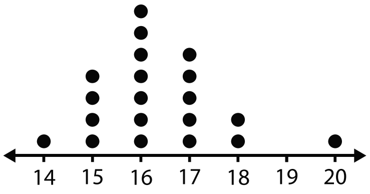
A dot plot is a graphic that summarizes a set of data.

Example: Consider the data set shown below,

20	16	18	15	17	17	16	16	18	16
17	15	16	17	16	16	15	14	15	17

In the list, the data is in an unorganized form. When the same data is organized as a dot plot, it is easier to see a few things. For example, you can see that 16 is the mode, the data is clustered around the mode, and there is a gap between 18 and 20.

In a dot plot, there is always a number line across the bottom. Above each number, there are dots and each dot represents an observation or data point. For example, the dot plot shown below may represent a class of 20 sixth graders. Each student scored between 14 and 20 points on a science fair project. The dot plot would show that only one student received a score of 20 and only one received a score of 14, while most students received a score of 15, 16, or 17.



Help Pages

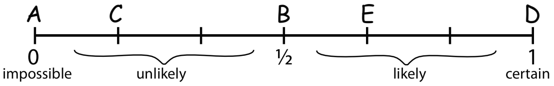
Statistics and Probability 7.SP.1 – 7.SP.8

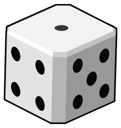
Probability

Probability is the likelihood that an event will occur. Express the probability of an event as a fraction between 0 and 1. When an event is certain, it has a probability of 1. When an event is impossible, it has a probability of zero.

Example: Consider rolling a number cube. Plot the possibility that each of the following outcomes would occur.

A) a number 7
B) any even number
C) a number 2
D) any number 1 through 6
E) a number greater than 2



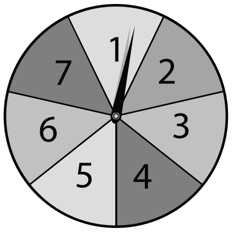


Explanation:

A) There is no 7 on a number cube, so *P*(7) is impossible.
B) There are six possible outcomes on the number cube and three of them are even numbers, so *P*(even number) is $\frac{1}{2}$.
C) The number 2 occurs one time on the number cube, so *P*(2) is $\frac{1}{6}$, or unlikely.
D) 1 through 6 includes all the possible outcomes so *P*(1,2,3,4,5 or 6) is certain.
E) There are four numbers greater than 2 on the number cube, so *P*(>2) is $\frac{4}{6}$, or likely.

Theoretical probability is the ratio of the number of ways an event can occur to the number of possible outcomes (or sample space).

$$\text{Theoretical Probability} \quad P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$



Example: Look at the spinner. What is the theoretical probability that Mark will spin an odd number?

Answer: There are 7 possible outcomes on the spinner. 4 of these outcomes are odd numbers. Therefore, the P(odd) = $\frac{4}{7}$.

Experimental probability (also called relative frequency) is what actually happens in a trial.

$$\text{Experimental Probability} \quad P(\text{event}) = \frac{\text{number of times the favorable event occurs}}{\text{total number of trials}}$$

Example: Mark spins the spinner ten times. The results are shown in the table below. Calculate the experimental probability.

Spin	1	2	3	4	5	6	7	8	9	10
Outcome	3	5	4	6	6	1	2	2	3	4

Answer: The total number of trials is ten. Four of these outcomes are odd numbers. Therefore, the *P*(odd) = $\frac{4}{10}$.

Experimental probability or relative frequency tends to get closer to theoretical probability as the number of trials increases. For example, the probability of getting heads on a coin toss is $\frac{1}{2}$ or 50%. In a trial of 100 tosses, heads may not come up exactly 50 times — it may be more like 48 or 53 times. However, if the trial included 10,000 tosses, the event (heads) would be much closer to 50%.

Help Pages

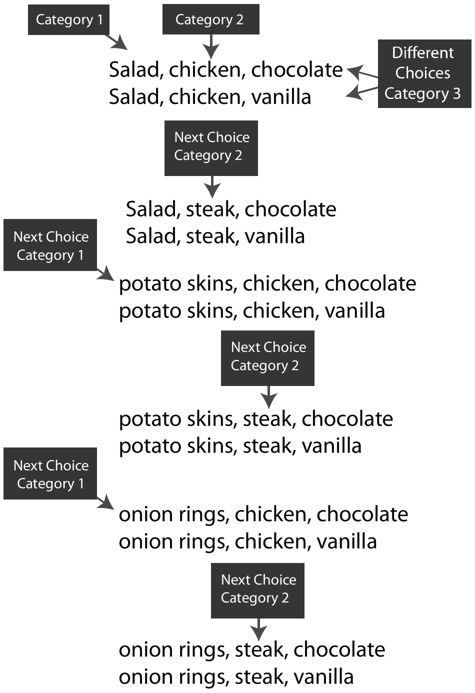
Statistics and Probability 7.SP.1 – 7.SP.8

Sample Space – Organized List

To find the probability of an event, first identify the sample space. **The sample space is the set of all possible outcomes for an experiment or exercise.**
One way to show sample space is to use an **organized list**.

Example: Larry is eating dinner at the Bountiful Buffet. The buffet provides three appetizers – salad, potato skins or onion rings. There are two choices for a main dish, as well: chicken or steak. Finally, Larry has two choices for dessert, chocolate or vanilla pudding. If Larry chooses one food from each category, how many possible combinations can Larry make?

1. To make an organized list, choose an item from the first category.
2. Now pair this item with one choice from the second category.
3. Next, pull in one choice from the third category.
4. Continue in this way until you've worked through all choices.
5. The completed list represents the **sample space**.
6. Count the options to find how many combinations Larry can make.



Salad, chicken, chocolate
Salad, chicken, vanilla

Salad, steak, chocolate
Salad, steak, vanilla

potato skins, chicken, chocolate
potato skins, chicken, vanilla

potato skins, steak, chocolate
potato skins, steak, vanilla

onion rings, chicken, chocolate
onion rings, chicken, vanilla

onion rings, steak, chocolate
onion rings, steak, vanilla

There are 12 possible combinations.

Sample Space – Tables

A **table** is another way to determine a sample space. To create a sample space using a table, list all of the possible outcomes for one category across the top of a table and all of the possible outcomes for the other category down the left side of the table. Complete the table as shown.
Example: Joan is preparing treats for her daughter's birthday party. She has three flavors of cake and three flavors of ice cream. Complete a table to show the sample space for all the possible combinations of cake and ice cream.

Sample Space

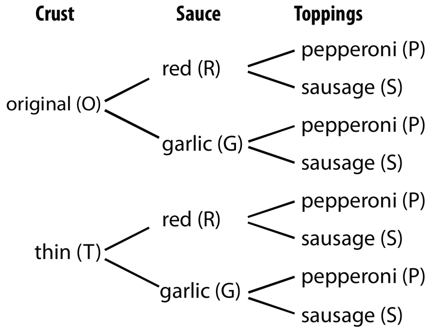
	Chocolate	**Yellow**	**Carrot**
Strawberry	chocolate strawberry	yellow strawberry	carrot strawberry
Vanilla	chocolate vanilla	yellow vanilla	carrot vanilla
Cherry	chocolate cherry	yellow cherry	carrot cherry

There are 9 possible combinations.

Sample Space – Tree Diagram

A **tree diagram** can also be used to show all the possible outcomes (sample space).

Example: For lunch on Friday's, the school cafeteria offers pizza. Students can choose from two types of crust — original or thin, two flavors of sauce — red or garlic, and two types of toppings — pepperoni or sausage. Use a tree diagram to show all possible outcomes.



There are 8 possible combinations.

Help Pages

Statistics and Probability 7.SP.1 – 7.SP.8

Fundamental Counting Principle

Like organized lists and trees, the **fundamental counting principle** is another way to determine the possible number of outcomes or combinations for a series of choices.

Example: Tom has three pairs of pants, four shirts, five ties and two pair of shoes. How many possible combinations of outfits does Tom have?

1. First determine the number of categories or events. In this problem there are four categories: pants, shirts, ties and shoes.

Pants	**Shirts**	**Ties**	**Shoes**
3	4	5	2

2. For each category, identify the number of choices.
3. Finally, multiply the number of choices to determine the number of possible combinations or outcomes.
4. $3 \times 4 \times 5 \times 2 = 120$ possible combinations.

Example: Lisa is filling out a survey with five questions. For each question, Lisa answers yes or no. How many possible outcomes does the survey have?

1. In this example, each question is a category, so there are five categories: Question 1, Question 2, Question 3, Question 4, Question 5
2. For each question, there are two choices.

Question 1	**Question 2**	**Question 3**	**Question 4**	**Question 5**
2	2	2	2	2

3. To find the total possible outcomes, we multiply $2 \times 2 \times 2 \times 2 \times 2 = 32$.

4. There are 32 possible outcomes for the survey.

Help Pages Index

Help Pages Index