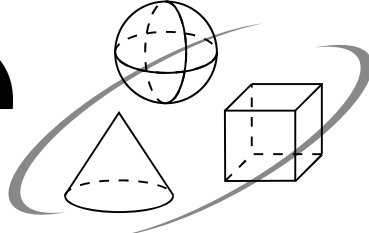


# Simple Solutions.



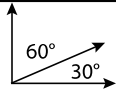
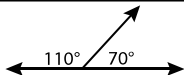
Minutes a Day—Mastery for a Lifetime!

## Pre-Algebra

**Help Pages &  
“Who Knows?”**









## Help Pages

## Vocabulary

General	
<b>absolute value</b>	— the distance between a number, $x$ , and zero on a number line; written as $ x $ . <b>Example:</b> $ 5  = 5$ reads "The absolute value of 5 is 5." $ -7  = 7$ reads "The absolute value of -7 is 7."
<b>composite number</b>	— a number with more than 2 factors. <b>Example:</b> 10 has factors of 1, 2, 5, and 10. Ten is a composite number.
<b>exponent</b>	— tells the number of times that a base is multiplied by itself. An exponent is written to the upper right of the base. <b>Example:</b> $5^3 = 5 \times 5 \times 5$ . The exponent is 3.
<b>expression</b>	— a mathematical phrase written in symbols. <b>Example:</b> $2x + 5$ is an expression.
<b>factors</b>	— are multiplied together to get a product. <b>Example:</b> 2 and 3 are factors of 6.
<b>greatest common factor (GCF)</b>	— the highest factor that 2 numbers have in common. <b>Example:</b> The factors of 6 are 1, 2, 3, and 6. The factors of 9 are 1, 3, and 9. The GCF of 6 and 9 is 3.
<b>integers</b>	— the set of whole numbers, positive or negative, and zero.
<b>least common multiple (LCM)</b>	— the smallest multiple that 2 numbers have in common. <b>Example:</b> Multiples of 3 are 3, 6, 9, 12, 15... Multiples of 4 are 4, 8, 12, 16... The LCM of 3 and 4 is 12.
<b>multiples</b>	— can be evenly divided by a number. <b>Example:</b> 5, 10, 15, and 20 are multiples of 5.
<b>prime factorization</b>	— a number, written as a product of its prime factors. <b>Example:</b> 140 can be written as $2 \times 2 \times 5 \times 7$ or $2^2 \times 5 \times 7$ .
<b>prime number</b>	— a number with exactly 2 factors (the number itself and 1). 1 is not prime (it has only 1 factor). <b>Example:</b> 7 has factors of 1 and 7. Seven is a prime number.
<b>square root</b>	— a number that when multiplied by itself gives you another number. The symbol for square root is $\sqrt{x}$ . <b>Example:</b> $\sqrt{49} = 7$ reads "The square root of 49 is 7."
<b>term</b>	— the components of an expression, usually being added to or subtracted from each other. <b>Example:</b> The expression $2x + 5$ has two terms: $2x$ and 5. The expression $3n^2$ has only one term.
<b>variable</b>	— a letter or symbol in an algebraic expression that represents a number.
Geometry	
<b>acute angle</b>	— an angle measuring less than $90^\circ$ .
<b>complementary angles</b>	— two angles whose measures add up to $90^\circ$ . 
<b>congruent</b>	— figures with the same shape and the same size.
<b>obtuse angle</b>	— an angle measuring more than $90^\circ$ .
<b>right angle</b>	— an angle measuring exactly $90^\circ$ .
<b>similar</b>	— figures having the same shape, but different size.
<b>straight angle</b>	— an angle measuring exactly $180^\circ$ .
<b>supplementary angles</b>	— two angles whose measures add up to $180^\circ$ . 
<b>surface area</b>	— the sum of the areas of all of the faces of a solid figure.

## Help Pages

## Vocabulary (continued)

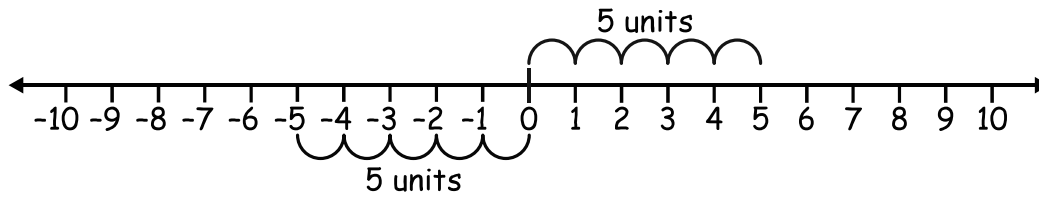
Geometry — Circles			
circumference — the distance around the outside of a circle.			
diameter — the widest distance across a circle. The diameter always passes through the center.			
radius — the distance from any point on the circle to the center. The radius is half of the diameter.			
Geometry — Polygons			
Number of Sides	Name	Number of Sides	Name
3 	triangle	7 	heptagon
4 	quadrilateral	8 	octagon
5 	pentagon	9 	nonagon
6 	hexagon	10 	decagon
Geometry — Triangles			
equilateral — a triangle in which all 3 sides have the same length.			
isosceles — a triangle in which 2 sides have the same length.			
scalene — a triangle in which no sides are the same length.			
Measurement — Relationships			
Volume		Distance	
3 teaspoons in a tablespoon		36 inches in a yard	
2 cups in a pint		1,760 yards in a mile	
2 pints in a quart		5,280 feet in a mile	
4 quarts in a gallon		100 centimeters in a meter	
Weight		1,000 millimeters in a meter	
16 ounces in a pound		Temperature	
2,000 pounds in a ton		0°Celsius - freezing point	
Time		100°Celsius - boiling point	
10 years in a decade		32°Fahrenheit - freezing point	
100 years in a century		212°Fahrenheit - boiling point	
Ratio and Proportion			
proportion — a statement that two ratios (or fractions) are equal. <b>Example:</b> $\frac{1}{2} = \frac{3}{6}$			
percent (%) — the ratio of any number to 100. <b>Example:</b> 14% means 14 out of 100 or $\frac{14}{100}$ .			

## Help Pages

## Solved Examples

## Absolute Value

The **absolute value** of a number is its distance from zero on a number line. It is always positive.

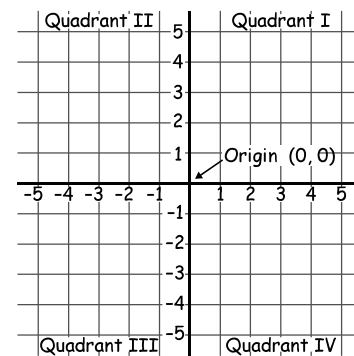
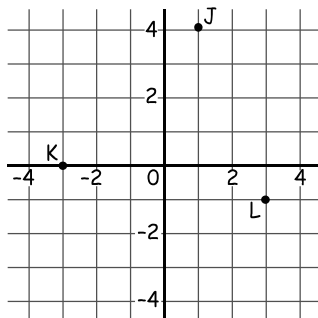


The absolute value of both  $-5$  and  $+5$  is  $5$ , because both are  $5$  units away from zero. The symbol for the absolute value of  $-5$  is  $|-5|$ . Examples:  $|-3| = 3$ ;  $|8| = 8$ .

## Coordinate Graphing

A **coordinate plane** is formed by the intersection of a horizontal number line, called the **x-axis**, and a vertical number line, called the **y-axis**. The axes meet at the point  $(0, 0)$ , called the **origin**, and divide the coordinate plane into four **quadrants**.

Points are represented by **ordered pairs** of numbers,  $(x, y)$ . The first number in an ordered pair is the **x-coordinate**; the second number is the **y-coordinate**. In the point  $(-4, 1)$ ,  $-4$  is the **x-coordinate** and  $1$  is the **y-coordinate**.



When graphing on a coordinate plane, always move on the **x-axis** first (right or left), and then move on the **y-axis** (up or down).

- The coordinates of point J are  $(1, 4)$ .
- The coordinates of point K are  $(-3, 0)$ .
- The coordinates of point L are  $(3, -1)$ .

## Decimals

**Adding or subtracting decimals** is very similar to adding or subtracting whole numbers. The main difference is that you have to line up the decimal points in the numbers before you begin.

**Examples:** Find the sum of  $3.14$  and  $1.2$ .

$$\begin{array}{r} 3.14 \\ + 1.20 \\ \hline 4.34 \end{array}$$

**Examples:** Subtract  $3.7$  from  $9.3$ .

$$\begin{array}{r} 9.3 \\ - 3.7 \\ \hline 5.6 \end{array}$$

Add  $55.1$ ,  $6.472$  and  $18.33$ .

$$\begin{array}{r} 55.100 \\ 6.472 \\ + 18.330 \\ \hline 79.902 \end{array}$$

Find the difference of  $4.1$  and  $2.88$ .

$$\begin{array}{r} 4.10 \\ - 2.88 \\ \hline 1.22 \end{array}$$

1. Line up the decimal points. Add zeroes as needed.
2. Add (or subtract) the decimals.
3. Add (or subtract) the whole numbers.
4. Bring the decimal point straight down.

# Help Pages

## Solved Examples

### Decimals (continued)

When multiplying a decimal by a whole number, the process is similar to multiplying whole numbers.

**Examples:** Multiply 3.42 by 4.

Find the product of 2.3 and 2.

$$\begin{array}{r} 3.42 \rightarrow 2 \text{ decimal places} \\ \times 4 \rightarrow 0 \text{ decimal places} \\ \hline \end{array}$$

13.68 → Place decimal point so there are 2 decimal places.

1. Line up the numbers on the right.
2. Multiply. Ignore the decimal point.
3. Place the decimal point in the product. (The total number of decimal places in the product must equal the total number of decimal places in the factors.)

$$\begin{array}{r} 2.3 \rightarrow 1 \text{ decimal place} \\ \times 2 \rightarrow 0 \text{ decimal places} \\ \hline \end{array}$$

4.6 → Place decimal point so there is 1 decimal place.

The process for multiplying two decimal numbers is a lot like the process described above.

**Examples:** Multiply 0.4 by 0.6.

Find the product of 2.67 and 0.3.

$$\begin{array}{r} 0.4 \rightarrow 1 \text{ decimal place} \\ \times 0.6 \rightarrow 1 \text{ decimal place} \\ \hline \end{array}$$

0.24 → Place decimal point so there are 2 decimal places.

$$\begin{array}{r} 2.67 \rightarrow 2 \text{ decimal places} \\ \times 0.3 \rightarrow 1 \text{ decimal place} \\ \hline \end{array}$$

0.801 → Place decimal point so there are 3 decimal places.

Sometimes it is necessary to add zeroes in the product as placeholders in order to have the correct number of decimal places.

**Example:** Multiply 0.03 by 0.4.

2 decimal places → 0.03

1 decimal place →  $\times 0.4$

Place decimal point so there are 3 decimal places. → 0.012

A zero had to be added in front of the 12, so there would be 3 decimal places in the product.

The process for dividing a decimal number by a whole number is similar to dividing whole numbers.

**Examples:** Divide 6.4 by 8.

Find the quotient of 20.7 and 3.

$$\begin{array}{r} 0.8 \\ 8 \overline{)6.4} \\ \underline{-6.4} \\ 0 \end{array}$$

1. Set up the problem for long division.
2. Place the decimal point in the quotient directly above the decimal point in the dividend.
3. Divide. Add zeroes as placeholders if necessary. (examples below)

$$\begin{array}{r} 6.9 \\ 3 \overline{)20.7} \\ \underline{-18} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

**Examples:** Divide 4.5 by 6.

Find the quotient of 3.5 and 4.

$$\begin{array}{r} 0.75 \\ 6 \overline{)4.50} \\ \underline{-4.2} \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

← Add a zero(es).

← Bring zero down. Keep dividing.

$$\begin{array}{r} 0.875 \\ 4 \overline{)3.500} \\ \underline{-3.2} \\ 30 \\ \underline{-28} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

## Help Pages

## Solved Examples

## Decimals (continued)

When dividing decimals, the remainder is not always zero. Sometimes, the division continues on and on and the remainder begins to repeat itself. This quotient is called a **repeating decimal**.

**Examples:** Divide 2 by 3.

$$\begin{array}{r} 0.6\overline{6} \\ 3 \overline{) 2.000} \\ \underline{-18} \phantom{00} \\ 20 \phantom{0} \\ \underline{-18} \phantom{0} \\ 20 \phantom{0} \end{array}$$

Divide 10 by 11.

$$\begin{array}{r} 0.90\overline{90} \\ 11 \overline{) 10.00000} \\ \underline{-99} \phantom{0000} \\ 100 \phantom{00} \\ \underline{-99} \phantom{00} \\ 100 \phantom{00} \end{array}$$

Add zeroes as needed

This pattern begins to repeat itself (with the same remainder).

To write the final answer, put a bar in the quotient over the digits that repeat.

The process for **dividing a decimal number by a decimal number** is similar to other long division that you have done. The main difference is that the decimal point has to be moved in both the dividend and the divisor the same number of places to the right.

**Example:** Divide 1.8 by 0.3.

$$\begin{array}{r} 6. \\ 0.3 \overline{) 1.8} \\ \underline{-18} \\ 0 \end{array}$$

1. Change the divisor to a whole number by moving the decimal point as many places to the right as possible.
2. Move the decimal in the dividend the same number of places to the right as you did in the divisor.
3. Put the decimal point in the quotient directly above the decimal point in the dividend. Divide.

Divide 0.385 by 0.05.

$$\begin{array}{r} 7.7 \\ 0.05 \overline{) 0.385} \\ \underline{-35} \phantom{0} \\ 35 \\ \underline{-35} \\ 0 \end{array}$$

## Equations

An equation consists of two expressions separated by an equal sign. You have worked with simple equations for a long time:  $2 + 3 = 5$ . More complicated equations involve variables which replace a number. To solve an equation like this, you must figure out which number the variable stands for. A simple example is when  $2 + x = 5$ . Here, the variable,  $x$ , stands for 3.

Sometimes an equation is not so simple. In these cases, there is a process for solving for the variable. No matter how complicated the equation, the goal is to work with the equation until all the numbers are on one side and the variable is alone on the other side. These equations will require only **one step** to solve. To check your answer, put the value of  $x$  back into the original equation.

Solving an equation with a variable on one side:

**Example:** Solve for  $x$ .  $x + 13 = 27$

$$\begin{array}{r} x + 13 = 27 \\ \underline{-13 = -13} \\ x = 14 \end{array}$$

**Example:** Solve for  $a$ .  $a - 22 = -53$

$$\begin{array}{r} a - 22 = -53 \\ \underline{+22 = +22} \\ a = -31 \end{array}$$

Check:  $-31 - 22 = -53$  ✓ correct!

1. Look at the side of the equation that has the variable on it. If there is a number added to or subtracted from the variable, it must be removed. In the first example, 13 is added to  $x$ .
2. To remove 13, add its opposite (-13) to both sides of the equation.
3. Add downward.  $x$  plus nothing is  $x$ . 13 plus -13 is zero. 27 plus -13 is 14.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of  $x$ .  $x = 14$ .

## Help Pages

## Solved Examples

## Equations (continued)

In the next examples, a number is either multiplied or divided by the variable (not added or subtracted).

**Example:** Solve for  $x$ .  $3x = 39$

$$3x = 39$$

$$\frac{3x}{3} = \frac{39}{3}$$

$$x = 13$$

Check:  $3(13) = 39$   
 $39 = 39$  ✓ correct!

**Example:** Solve for  $n$ .  $\frac{n}{6} = -15$

$$\frac{n}{6}(\cancel{6}) = -15(6)$$

$$n = -90$$

Check:  $\frac{-90}{6} = -15$

$-15 = -15$  ✓ correct!

1. Look at the side of the equation that has the variable on it. If there is a number multiplied by or divided into the variable, it must be removed. In the first example, 3 is multiplied by  $x$ .
2. To remove 3, divide both sides by 3. (You divide because it is the opposite operation from the one in the equation (multiplication)).
3. Follow the rules for multiplying or dividing integers.  $3x$  divided by 3 is  $x$ .  $39$  divided by 3 is thirteen.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of  $x$ .  $x = 13$ .

The next set of examples also have a variable on only one side of the equation. These, however, are a bit more complicated, because they will require **two steps** in order to get the variable alone.

**Example:** Solve for  $x$ .  $2x + 5 = 13$

$$2x + 5 = 13$$

$$\frac{-5}{-5} = \frac{-5}{-5}$$

$$2x = 8$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$$

$$x = 4$$

Check:  $2(4) + 5 = 13$   
 $8 + 5 = 13$   
 $13 = 13$  ✓ correct!

**Example:** Solve for  $n$ .

$$3n - 7 = 32$$

$$\frac{+7}{+7} = \frac{+7}{+7}$$

$$3n = 39$$

$$\frac{\cancel{3}n}{\cancel{3}} = \frac{39}{3}$$

$$n = 13$$

Check:  $3(13) - 7 = 32$

$$39 - 7 = 32$$

$32 = 32$  ✓ correct!

1. Look at the side of the equation that has the variable on it. There is a number (2) multiplied by the variable, and there is a number added to it (5). Both of these must be removed. Always begin with the addition/subtraction. To remove the 5 we must add its opposite (-5) to both sides.
2. To remove the 2, divide both sides by 2. (You divide because it is the opposite operation from the one in the equation (multiplication)).
3. Follow the rules for multiplying or dividing integers.  $2x$  divided by 2 is  $x$ .  $8$  divided by 2 is four.
4. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of  $x$ .  $x = 4$ .

## Help Pages

## Solved Examples

## Equations (continued)

These multi-step equations also have a variable on only one side. To get the variable alone, though, requires several steps.

**Example:** Solve for  $x$ .  $3(2x + 3) = 21$

$$\cancel{3} \left( \frac{2x + 3}{\cancel{3}} \right) = \frac{21}{3}$$

$$\begin{array}{r} 2x + 3 = 7 \\ -3 = -3 \\ \hline 2x = 4 \\ \cancel{2} x = \frac{4}{\cancel{2}} \\ x = 2 \end{array}$$

**Check:**  $3(2(2) + 3) = 21$   
 $3(4 + 3) = 21$   
 $3(7) = 21$   
 $21 = 21 \checkmark$  correct!

1. Look at the side of the equation that has the variable on it. First, the expression  $(2x + 3)$  is multiplied by 3; then there is a number (3) added to  $2x$ , and there is a number (2) multiplied by  $x$ . All of these must be removed. To remove the 3 outside the parentheses, divide both sides by 3. (You divide because it is the opposite operation from the one in the equation (multiplication).
2. To remove the 3 inside the parentheses, add its opposite  $(-3)$  to both sides.
3. Remove the 2 by dividing both sides by 2.
4. Follow the rules for multiplying or dividing integers.  $2x$  divided by 2 is  $x$ .  $4$  divided by 2 is two.
5. Once the variable is alone on one side of the equation, the equation is solved. The bottom line tells the value of  $x$ .  $x = 2$ .

When solving an **equation with a variable on both sides**, the goals are the same: to get the numbers on one side of the equation and to get the variable alone on the other side.

**Example:** Solve for  $x$ .  $2x + 4 = 6x - 4$

$$\begin{array}{r} 2x + 4 = 6x - 4 \\ -2x \quad = -2x \\ \hline 4 = 4x - 4 \\ +4 = \quad +4 \\ \hline 8 = 4x \\ \frac{8}{4} = \frac{4x}{4} \\ 2 = x \end{array}$$

**Check:**  $2(2) + 4 = 6(2) - 4$   
 $4 + 4 = 12 - 4$   
 $8 = 8 \checkmark$  correct!

1. Since there are variables on both sides, the first step is to remove the "variable term" from one of the sides by adding its opposite. To remove  $2x$  from the left side, add  $-2x$  to both sides.
2. There are also numbers added (or subtracted) to both sides. Next, remove the number added to the variable side by adding its opposite. To remove  $-4$  from the right side, add  $+4$  to both sides.
3. The variable still has a number multiplied by it. This number (4) must be removed by dividing both sides by 4.
4. The final line shows that the value of  $x$  is 2.



## Help Pages

## Solved Examples

## Equations (continued)

**Example:** Solve for  $n$ .  $5n - 3 = 8n + 9$

Check:  $5(-4) - 3 = 8(-4) + 9$   
 $-20 - 3 = -32 + 9$   
 $-23 = -23$  ✓ correct!

$$\begin{array}{r} 5n - 3 = 8n + 9 \\ -8n \quad = -8n \\ \hline -3n - 3 = 9 \\ +3 = +3 \\ \hline -3n = 12 \\ \hline \cancel{3}n = 12 \\ \cancel{3} \quad = -3 \\ n = -4 \end{array}$$

## Exponents

An **exponent** is a small number to the upper right of another number (the base). Exponents are used to show that the base is a repeated factor.

**Example:**  $2^4$  is read "two to the fourth power."

The base (2) is a factor multiple times.

The exponent (4) tells how many times the base is a factor.

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

base  $\rightarrow$   $2^4$   $\leftarrow$  exponent

**Example:**  $9^3$  is read "nine to the third power" and means  $9 \times 9 \times 9 = 729$

## Expressions

An **expression** is a number, a variable, or any combination of these, along with operation signs (+, -, ×, ÷) and grouping symbols. An expression never includes an equal sign.

Five examples of expressions are 5,  $x$ ,  $(x + 5)$ ,  $(3x + 5)$ , and  $(3x^2 + 5)$ .

To **evaluate an expression** means to calculate its value using specific variable values.

**Example:** Evaluate  $2x + 3y + 5$  when  $x = 2$  and  $y = 3$ .

$$2(2) + 3(3) + 5 = ?$$

$$4 + 9 + 5 = ?$$

$$13 + 5 = 18$$

The expression has a value of 18.

1. To evaluate, put the values of  $x$  and  $y$  into the expression.
2. Use the rules for integers to calculate the value of the expression.

**Example:** Find the value of  $\frac{xy}{3} + 2$  when  $x = 6$  and  $y = 4$ .

$$\frac{6(4)}{3} + 2 = ?$$

$$\frac{24}{3} + 2 = ?$$

$$8 + 2 = 10$$

The expression has a value of 10.

## Help Pages

## Solved Examples

## Expressions (continued)

When evaluating a numerical expression containing multiple operations, use a set of rules called the **Order of Operations**. The Order of Operations determines which operations, and in which order, they should be performed. (Which operation should be done first, second, etc.)

The Order of Operations is as follows:

1. Parentheses
2. Exponents
3. Multiplication/Division (left to right in the order that they occur)
4. Addition/Subtraction (left to right in the order that they occur)

If parentheses are enclosed within other parentheses, work from the inside out.

To remember the order, use the mnemonic device "Please **Excuse My Dear Aunt Sally**."

Use the following examples to help you understand how to use the Order of Operations.

**Example:**  $2 + 6 \cdot 5$

To evaluate this expression, work through the steps using the Order of Operations.

1. Since there are no parentheses or exponents in the expression, skip steps 1 and 2.
2. According to step 3, do multiplication and division.  $6 \cdot 5 = 30$
3. Next, step 4 says to do addition and subtraction.  $2 + 30 = 32$

The answer is 32.

**Example:**  $42 \div 6 \cdot 3 + 4 - 16 \div 2$

$$42 \div 6 \cdot 3 + 4 - 16 \div 2$$

$$7 \cdot 3 + 4 - 16 \div 2$$

$$21 + 4 - 16 \div 2$$

$$21 + 4 - 8$$

$$25 - 8$$

$$17$$

1. Do multiplication and division first (in the order they occur).

2. Do addition and subtraction next (in the order they occur).

**Example:**  $42 \div 6 \cdot 3 + 4 - 16 \div 2$

$$5(2 + 4) + 15 \div (9 - 6)$$

$$5(6) + 15 \div (3)$$

$$30 + 5$$

$$35$$

1. Do operations inside of parentheses first.

2. Do multiplication and division first (in the order they occur).

3. Do addition and subtraction next (in the order they occur).

**Example:**  $4[3 + 2(7 + 5) - 7]$

$$4[3 + 2(7 + 5) - 7]$$

$$4[3 + 2(12) - 7]$$

$$4[3 + 24 - 7]$$

$$4[27 - 7]$$

$$4[20]$$

$$80$$

1. Brackets are treated as parentheses. Start from the innermost parentheses first.

2. Then work inside the brackets.

## Help Pages

## Solved Examples

## Expressions (continued)

Some expressions can be made more simple. There are a few processes for **simplifying an expression**. Deciding which process or processes to use depends on the expression itself. With practice, you will be able to recognize which of the following processes to use.

The **distributive property** is used when one term is multiplied by (or divided into) an expression that includes either addition or subtraction.  $a(b + c) = ab + ac$  or  $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$

**Example:** Simplify.  $3(2x + 5)$

$$\begin{aligned} & \overset{\curvearrowright}{\curvearrowleft} 3(2x + 5) = \\ & 3(2x) + 3(5) = \\ & 6x + 15 \end{aligned}$$

**Example:** Simplify.  $2(7x - 3y + 4)$

$$\begin{aligned} & \overset{\curvearrowright}{\curvearrowleft} 2(7x - 3y + 4) = \\ & 2(7x) + 2(-3y) + 2(+4) = \\ & 14x - 6y + 8 \end{aligned}$$

1. Since the 3 is multiplied by the expression,  $2x + 5$ , the 3 must be multiplied by both terms in the expression.
2. Multiply 3 by  $2x$  and then multiply 3 by  $+5$ .
3. The result includes both of these:  $6x + 15$ . Notice that simplifying an expression does not result in a single number answer, only a more simple expression.

Expressions which contain like terms can also be simplified. **Like terms** are those that contain the same variable to the same power.  $2x$  and  $-4x$  are like terms;  $3n^2$  and  $8n^2$  are like terms;  $5y$  and  $y$  are like terms; 3 and 7 are like terms.

An expression sometimes begins with like terms. This process for simplifying expressions is called **combining like terms**. When combining like terms, first identify the like terms. Then, simply add the like terms to each other and write the results together to form a new expression.

**Example:** Simplify.  $2x + 5y - 9 + 5x - 3y - 2$

The like terms are  $2x$  and  $+5x$ ,  
 $+5y$  and  $-3y$ , and  $-9$  and  $-2$ .

$2x + +5x = +7x$ ,  $+5y + -3y = +2y$ ,  
and  $-9 + -2 = -11$ .

The result is  $7x + 2y - 11$ .

The next examples are a bit more complex. It is necessary to use the distributive property first, and then to combine like terms.

**Example:** Simplify.  $2(3x + 2y + 2) + 3(2x + 3y + 2)$

$$\begin{array}{r} 6x + 4y + 4 \\ +6x + 9y + 6 \\ \hline 12x + 13y + 10 \end{array}$$

**Example:** Simplify.  $4(3x - 5y - 4) - 2(3x - 3y + 2)$

$$\begin{array}{r} +12x - 20y - 16 \\ -6x + 6y - 4 \\ \hline 6x - 14y - 20 \end{array}$$

1. First, apply the distributive property to each expression. Write the results on top of each other, lining up the like terms with each other. Pay attention to the signs of the terms.
2. Then, add each group of like terms. Remember to follow the rules for integers.

## Help Pages

### Solved Examples

#### Expressions (continued)

Other expressions that can be simplified are written as fractions. **Simplifying** these expressions (**algebraic fractions**) is similar to simplifying numerical fractions. It involves cancelling out factors that are common to both the numerator and the denominator.

Simplify.  $\frac{12x^2yz^4}{16xy^3z^2}$

$$\frac{\overset{3}{\cancel{12}} \overset{x}{\cancel{x^2}} \cancel{y} \overset{z^2}{\cancel{z^4}}}{\underset{4}{\cancel{16}} \cancel{x} \overset{y^2}{\cancel{y^3}} \cancel{z^2}}$$

$$\frac{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{x} \cdot x \cdot \cancel{y} \cdot \cancel{z} \cdot \cancel{z} \cdot z \cdot z}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot \cancel{x} \cdot \cancel{y} \cdot y \cdot y \cdot \cancel{z} \cdot \cancel{z}}$$

$$\frac{3xz^2}{4y^2}$$

1. Begin by looking at the numerals in both the numerator and denominator (12 and 16). What is the largest number that goes into both evenly? Cancel this factor (4) out of both.
2. Look at the  $x$  portion of both numerator and denominator. What is the largest number of  $x$ 's that can go into both of them? Cancel this factor ( $x$ ) out of both.
3. Do the same process with  $y$  and then  $z$ . Cancel out the largest number of each ( $y$  and  $z^2$ ). Write the numbers that remain in the numerator or denominator for your answer.

Often a relationship is described using verbal (English) phrases. In order to work with the relationship, you must first **translate it into an algebraic expression or equation**. In most cases, word clues will be helpful. Some examples of verbal phrases and their corresponding algebraic expressions or equations are written below.

<u>Verbal Phrase</u>	<u>Algebraic Expression</u>
Ten more than a number .....	$x + 10$
The sum of a number and five .....	$x + 5$
A number increased by seven .....	$x + 7$
Six less than a number .....	$x - 6$
A number decreased by nine .....	$x - 9$
The difference between a number and four .....	$x - 4$
The difference between four and a number .....	$4 - x$
Five times a number .....	$5x$
Eight times a number, increased by one .....	$8x + 1$
The product of a number and six is twelve. ....	$6x = 12$
The quotient of a number and 10 .....	$\frac{x}{10}$
The quotient of a number and two, decreased by five .....	$\frac{x}{2} - 5$

In most problems, the word "is" tells you to put in an equal sign. When working with fractions and percents, the word "of" generally means multiply. Look at the example below.

One half of a number is fifteen.

You can think of it as "one half times a number equals fifteen."

When written as an algebraic equation, it is  $\frac{1}{2}x = 15$ .

## Help Pages

## Solved Examples

## Expressions (continued)

At times you need to find the **greatest common factor (GCF)** of an algebraic expression.

**Example:** Find the GCF of  $12x^2yz^3$  and  $18xy^3z^2$ .

1. First, find the GCF of the numbers (12 and 18).  
The largest number that is a factor of both is **6**.
2. Now look at the  $x$ 's. Of the  $x$ -terms, which contains fewer  $x$ 's? Comparing  $x^2$  and  $x$ ,  $x$  has fewer.
3. Now look at the  $y$ 's and then the  $z$ 's. Again, of the  $y$ -terms,  $y$  contains fewer. Of the  $z$ -terms,  $z^2$  has fewer.
4. The GCF contains all of these.  **$6xyz^2$**

 $12x^2yz^3$  and  $18xy^3z^2$ 

The GCF of 12 and 18 is **6**.

Of  $x^2$  and  $x$ , the smaller is  $x$ .

Of  $y$  and  $y^3$ , the smaller is  $y$ .

Of  $z^3$  and  $z^2$ , the smaller is  $z^2$ .

The GCF is  $6xyz^2$ .

At other times you need to know the **least common multiple (LCM)** of an algebraic expression.

**Example:** Find the LCM of  $10a^3b^2c^2$  and  $15ab^4c$ .

1. First, find the LCM of the numbers (10 and 15).  
The lowest number that both go into evenly is **30**.
2. Now look at the  $a$ -terms. Which has more  $a$ 's?  
Comparing  $a^3$  and  $a$ ,  $a^3$  has more.
3. Now look at the  $b$ 's and then the  $c$ 's. Again, of the  $b$ -terms,  $b^4$  contains more. Of the  $c$ -terms,  $c^2$  contains more.
4. The LCM contains all of these.  **$30a^3b^4c^2$**

 $10a^3b^2c^2$  and  $15ab^4c$ 

The LCM of 10 and 15 is **30**.

Of  $a^3$  and  $a$ , the larger is  $a^3$ .

Of  $b^2$  and  $b^4$ , the larger is  $b^4$ .

Of  $c^2$  and  $c$ , the larger is  $c^2$ .

The LCM is  $30a^3b^4c^2$ .

## Fractions

When **adding fractions that have different denominators**, you first need to change the fractions so they have a common denominator. Then, you can add them.

Finding the **least common denominator (LCD)**: The LCD of the fractions is the same as the least common multiple of the denominators. Sometimes, the LCD will be the product of the denominators.

**Example:** Find the sum of  $\frac{3}{8}$  and  $\frac{1}{12}$ .

$$\begin{array}{r} 2 \overline{) 8, 12} \\ 2 \overline{) 4, 6} \\ 2 \overline{) 2, 3} \\ 3 \overline{) 1, 3} \\ \quad 1, 1 \end{array} \qquad \begin{array}{r} \frac{3}{8} = \frac{9}{24} \\ + \frac{1}{12} = \frac{2}{24} \\ \hline \frac{11}{24} \end{array}$$

$2 \times 2 \times 2 \times 3 = 24$   
The LCM is 24.

**Example:** Add  $\frac{1}{4}$  and  $\frac{1}{5}$ .

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ + \frac{1}{5} = \frac{4}{20} \\ \hline \frac{9}{20} \end{array}$$

$4 \times 5 = 20$   
The LCM is 20.

1. First, find the LCM of 8 and 12.
2. The LCM of 8 and 12 is 24. This is also the LCD of these 2 fractions.
3. Find an equivalent fraction for each that has a denominator of 24.
4. When they have a common denominator, the fractions can be added.

## Help Pages

## Solved Examples

## Fractions (continued)

When adding mixed numbers with unlike denominators, follow a process similar to the one you used with fractions (above). Be sure to put your answer in simplest form.

**Example:** Find the sum of  $6\frac{3}{7}$  and  $5\frac{2}{3}$ .

$$\begin{array}{r} 6\frac{3}{7} = 6\frac{9}{21} \\ + 5\frac{2}{3} = 5\frac{14}{21} \\ \hline 11\frac{23}{21} \end{array} \quad \frac{23}{21} = 1\frac{2}{21} + 11 = 12\frac{2}{21}$$

(improper)

1. Find the LCD.
2. Find the missing numerators.
3. Add the whole numbers, then add the fractions.
4. Make sure your answer is in simplest form.

When subtracting numbers with unlike denominators, follow a process similar to the one you used when adding fractions. Be sure to put your answer in simplest form.

**Examples:** Find the difference of  $\frac{3}{4}$  and  $\frac{2}{5}$ .

$$\begin{array}{r} \frac{3}{4} = \frac{15}{20} \\ - \frac{2}{5} = \frac{8}{20} \\ \hline \frac{7}{20} \end{array}$$

1. Find the LCD just as you did when adding fractions.
2. Find the missing numerators.
3. Subtract the numerators and keep the common denominator.
4. Make sure your answer is in simplest form.

Subtract  $\frac{1}{16}$  from  $\frac{3}{8}$ .

$$\begin{array}{r} \frac{3}{8} = \frac{6}{16} \\ - \frac{1}{16} = \frac{1}{16} \\ \hline \frac{5}{16} \end{array}$$

When subtracting mixed numbers with unlike denominators, follow a process similar to the one you used when adding mixed numbers. Be sure to put your answer in simplest form.

**Example:** Subtract  $4\frac{2}{5}$  from  $8\frac{9}{10}$ .

$$\begin{array}{r} 8\frac{9}{10} = 8\frac{9}{10} \\ - 4\frac{2}{5} = 4\frac{4}{10} \\ \hline 4\frac{5}{10} = 4\frac{1}{2} \end{array}$$

1. Find the LCD.
2. Find the missing numerators.
3. Subtract and simplify your answer.

Sometimes when subtracting mixed numbers you may need to rename them. If the numerator of the top fraction is smaller than the numerator of the bottom fraction, rename the mixed number as in the example below.

**Example:** Subtract  $5\frac{5}{6}$  from  $9\frac{1}{4}$ .

1. Find the LCD.
2. Find the missing numerators. Write equivalent fractions.
3. Because you can't subtract 10 from 3, rename the whole number as a mixed number using the common denominator.
4. Add the two fractions to get an improper fraction.
5. Subtract the whole numbers and the fractions and simplify your answer.

$$\begin{array}{r} 9\frac{1}{4} = 9\frac{3}{12} = 8\frac{12}{12} + \frac{3}{12} = 8\frac{15}{12} \\ - 5\frac{5}{6} = 5\frac{10}{12} \\ \hline 3\frac{5}{12} \end{array}$$

## Help Pages

## Solved Examples

## Fractions (continued)

More examples of subtracting mixed numbers with unlike denominators:

$$8\frac{1}{2} = 8\frac{2}{4} = 7\frac{4}{4} + \frac{2}{4} = 7\frac{6}{4}$$

$$10\frac{1}{5} = 10\frac{4}{20} = 9\frac{20}{20} + \frac{4}{20} = 9\frac{24}{20}$$

$$\begin{array}{r} -4\frac{3}{4} = 4\frac{3}{4} = \\ \hline \end{array} \quad \begin{array}{r} -4\frac{3}{4} \\ \hline 3\frac{3}{4} \end{array}$$

$$\begin{array}{r} -6\frac{3}{4} = 6\frac{15}{20} = \\ \hline \end{array} \quad \begin{array}{r} -6\frac{15}{20} \\ \hline 3\frac{9}{20} \end{array}$$

To **multiply fractions**, simply multiply the numerators together to get the numerator of the product. Then multiply the denominators together to get the denominator of the product. Make sure your answer is in simplest form.

**Examples:** Multiply  $\frac{3}{5}$  by  $\frac{2}{3}$ .

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$$

1. Multiply the numerators.
2. Multiply the denominators.
3. Simplify your answer.

Multiply  $\frac{5}{8}$  by  $\frac{4}{5}$ .

$$\frac{5}{8} \times \frac{4}{5} = \frac{20}{40} = \frac{1}{2}$$

Sometimes you can use cancelling when multiplying fractions. Let's look at the examples again.

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{2}{\underset{1}{\cancel{3}}} = \frac{2}{5}$$

The 3s have a common factor — 3. Divide both of them by 3. Since,  $3 \div 3 = 1$ , we cross out the 3s and write 1s in their place.

Now, multiply the fractions. In the numerator,  $1 \times 2 = 2$ . In the denominator,  $5 \times 1 = 5$ .

The answer is  $\frac{2}{5}$ .

1. Are there any numbers in the numerator and the denominator that have common factors?
2. If so, cross out the numbers, divide both by that factor, and write the quotient.
3. Then, multiply the fractions as described above, using the quotients instead of the original numbers.

$$\frac{\overset{1}{\cancel{5}}}{2\cancel{8}} \times \frac{\underset{1}{\cancel{4}}}{\cancel{5}} = \frac{1}{2}$$

As in the other example, the 5s can be cancelled.

But here, the 4 and the 8 also have a common factor (4).

$8 \div 4 = 2$  and  $4 \div 4 = 1$ .

After cancelling both of these, you can multiply the fractions.

**REMEMBER:** You can cancel up and down or diagonally, but NEVER sideways!

When **multiplying mixed numbers**, you must first change them into improper fractions.

**Examples:** Multiply  $2\frac{1}{4}$  by  $3\frac{1}{9}$ .

$$2\frac{1}{4} \times 3\frac{1}{9} =$$

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{7}{\cancel{28}}}{\underset{1}{\cancel{9}}} = \frac{7}{1} = 7$$

1. Change each mixed number to an improper fraction.
2. Cancel wherever you can.
3. Multiply the fractions.
4. Put your answer in simplest form.

Multiply  $3\frac{1}{8}$  by 4.

$$3\frac{1}{8} \times 4 =$$

$$\frac{\overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{1} = \frac{25}{2} = 12\frac{1}{2}$$

## Help Pages

## Solved Examples

## Fractions (continued)

To **divide fractions**, you must take the reciprocal of the 2<sup>nd</sup> fraction, and then multiply that reciprocal by the 1<sup>st</sup> fraction. Don't forget to simplify your answer!

Examples: Divide  $\frac{1}{2}$  by  $\frac{7}{12}$ .

$$\frac{1}{2} \div \frac{7}{12} =$$

$$\frac{1}{2} \times \frac{12^6}{7} = \frac{6}{7}$$

1. Keep the 1<sup>st</sup> fraction as it is.
2. Write the reciprocal of the 2<sup>nd</sup> fraction.
3. Change the sign to multiplication.
4. Cancel if you can and multiply.
5. Simplify your answer.

Divide  $\frac{7}{8}$  by  $\frac{3}{4}$ .

$$\frac{7}{8} \div \frac{3}{4} =$$

$$\frac{7}{8} \times \frac{4^1}{3} = \frac{7}{6} = 1\frac{1}{6}$$

When **dividing mixed numbers**, you must first change them into improper fractions.

Example: Divide  $1\frac{1}{4}$  by  $3\frac{1}{2}$ .

$$1\frac{1}{4} \div 3\frac{1}{2} =$$

$$\frac{5}{4} \div \frac{7}{2} =$$

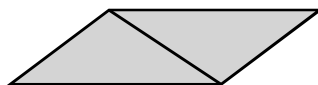
$$\frac{5}{4} \times \frac{2^1}{7} = \frac{5}{14}$$

1. Change each mixed number to an improper fraction.
2. Keep the 1<sup>st</sup> fraction as it is.
3. Write the reciprocal of the 2<sup>nd</sup> fraction.
4. Change the sign to multiplication.
5. Cancel if you can and multiply.
6. Simplify your answer.

## Geometry

To find the **area of a triangle**, first recognize that any triangle is exactly half of a parallelogram.

The whole figure is  
a parallelogram.

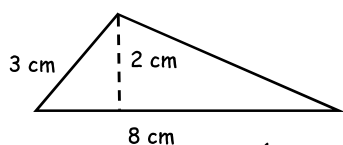


Half of the whole figure  
is a triangle.

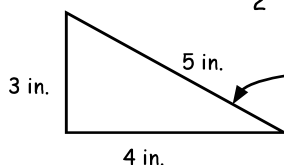
So, the triangle's area is equal to half of the product of the base and the height.

$$\text{Area of triangle} = \frac{1}{2}(\text{base} \times \text{height}) \quad \text{or} \quad A = \frac{1}{2}bh$$

Examples: Find the area of the triangles below.



$$\text{So, } A = 8 \text{ cm} \times 2 \text{ cm} \times \frac{1}{2} = 8 \text{ cm}^2$$



$$\text{So, } A = 4 \text{ in.} \times 3 \text{ in.} \times \frac{1}{2} = 6 \text{ in}^2.$$

1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down - never slanted.)
3. Multiply them together and divide by 2 to find the area. (8 cm<sup>2</sup>)

The base of this triangle is 4 inches long. Its height is 3 inches. (Remember the height is always straight up and down!)



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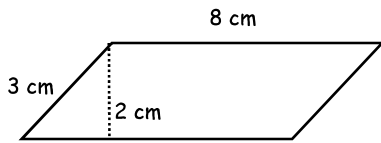
## Solved Examples

## Geometry

Finding the **area of a parallelogram** is similar to finding the area of any other quadrilateral. The area of the figure is equal to the length of its base multiplied by the height of the figure.

Area of parallelogram = base  $\times$  height or  $A = b \times h$

**Example:** Find the area of the parallelogram below.

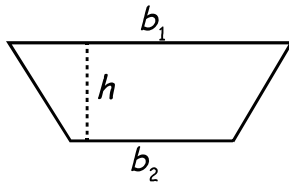


1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down - never slanted.)
3. Multiply to find the area. (16 cm<sup>2</sup>)

So,  $A = 8 \text{ cm} \times 2 \text{ cm} = 16 \text{ cm}^2$ .

Finding the **area of a trapezoid** is a little different than the other quadrilaterals that we have seen. Trapezoids have 2 bases of unequal length. To find the area, first find the average of the lengths of the 2 bases. Then, multiply that average by the height.

Area of trapezoid =  $\frac{\text{base}_1 + \text{base}_2}{2} \times \text{height}$  or  $A = \left(\frac{b_1 + b_2}{2}\right)h$

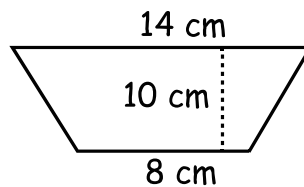


The bases are labeled  $b_1$  and  $b_2$ .

The height,  $h$ , is the distance between the bases.

**Example:** Find the area of the trapezoid below.

1. Add the lengths of the two bases. (22 cm)
2. Divide the sum by 2. (11 cm)
3. Multiply that result by the height to find the area. (110 cm<sup>2</sup>)



$$\frac{14 \text{ cm} + 8 \text{ cm}}{2} = \frac{22 \text{ cm}}{2} = 11 \text{ cm}$$

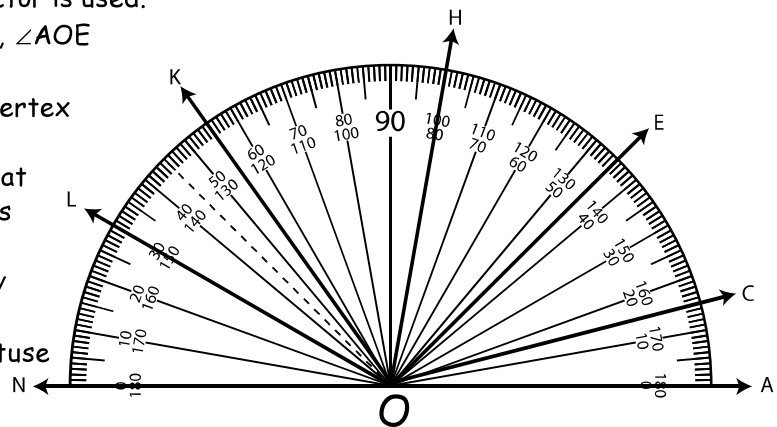
$$11 \text{ cm} \times 10 \text{ cm} = 110 \text{ cm}^2 = \text{Area}$$

To **find the measure of an angle**, a protractor is used.

The symbol for angle is  $\angle$ . On the diagram,  $\angle AOE$  has a measure less than  $90^\circ$ , so it is acute.

With the center of the protractor on the vertex of the angle (where the 2 rays meet), place one ray ( $\overline{OA}$ ) on one of the "0" lines. Look at the number that the other ray ( $\overline{OE}$ ) passes through. Since the angle is acute, use the lower set of numbers. Since  $\overline{OE}$  is halfway between the 40 and the 50, the measure of  $\angle AOE$  is  $45^\circ$ . (If it were an obtuse angle, the higher set of numbers would be used.)

Look at  $\angle NOH$ . It is an obtuse angle, so the higher set of numbers will be used. Notice that  $\overline{ON}$  is on the "0" line.  $\overline{OH}$  passes through the 100 mark. So the measure of  $\angle NOH$  is  $100^\circ$ .



## Help Pages

## Solved Examples

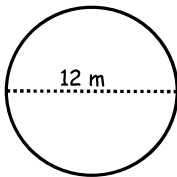
## Geometry (continued)

The **circumference of a circle** is the distance around the outside of the circle. Before you can find the circumference of a circle you must know either its radius or its diameter. Also, you must know the value of the constant, pi ( $\pi$ ).  $\pi = 3.14$  (rounded to the nearest hundredth)

Once you have this information, the circumference can be found by multiplying the diameter by pi.

$$\text{Circumference} = \pi \times \text{diameter} \quad \text{or} \quad C = \pi d$$

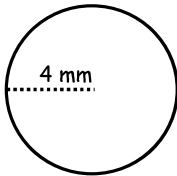
**Examples:** Find the circumference of the circles below.



1. Find the length of the diameter. (12 m)
2. Multiply the diameter by  $\pi$ . ( $12 \text{ m} \times 3.14$ )
3. The product is the circumference. (37.68 m)

$$\text{So, } C = 12 \text{ m} \times 3.14 = 37.68 \text{ m.}$$

Sometimes the radius of a circle is given instead of the diameter. Remember, the radius of any circle is exactly half of the diameter. If a circle has a radius of 3 feet, its diameter is 6 feet.



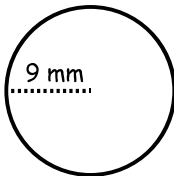
1. Since the radius is 4 mm, the diameter must be 8 mm.
2. Multiply the diameter by  $\pi$ . ( $8 \text{ mm} \times 3.14$ )
3. The product is the circumference. (25.12 mm)

$$\text{So, } C = 8 \text{ mm} \times 3.14 = 25.12 \text{ mm.}$$

When finding the **area of a circle**, the length of the radius is squared (multiplied by itself), and then that answer is multiplied by the constant, pi ( $\pi$ ).  $\pi = 3.14$  (rounded to the nearest hundredth).

$$\text{Area} = \pi \times \text{radius} \times \text{radius} \quad \text{or} \quad A = \pi r^2$$

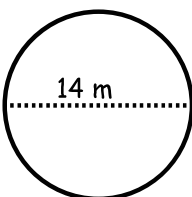
**Examples:** Find the area of the circles below.



1. Find the length of the radius. (9 mm)
2. Multiply the radius by itself. ( $9 \text{ mm} \times 9 \text{ mm}$ )
3. Multiply the product by pi. ( $81 \text{ mm}^2 \times 3.14$ )
4. The result is the area. (254.34  $\text{mm}^2$ )

$$\text{So, } A = 9 \text{ mm} \times 9 \text{ mm} \times 3.14 = 254.34 \text{ mm}^2.$$

Sometimes the diameter of a circle is given instead of the radius. Remember, the diameter of any circle is exactly twice the radius. If a circle has a diameter of 6 feet, its radius is 3 feet.



1. Since the diameter is 14 m, the radius must be 7 m.
2. Square the radius. ( $7 \text{ m} \times 7 \text{ m}$ )
3. Multiply that result by  $\pi$ . ( $49 \text{ m}^2 \times 3.14$ )
4. The product is the area. (153.86  $\text{m}^2$ )

$$\text{So, } A = (7 \text{ m})^2 \times 3.14 = 153.86 \text{ m}^2.$$

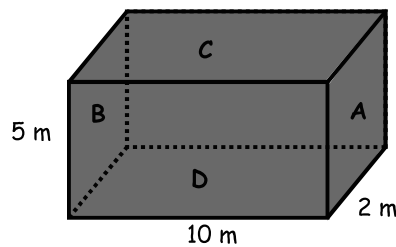
## Help Pages

## Solved Examples

## Geometry (continued)

To find the **surface area** of a solid figure, it is necessary to first count the total number of faces. Then, find the area of each of the faces; finally, add the areas of each face. That sum is the surface area of the figure.

Here, the focus will be on finding the **surface area of a rectangular prism**. A rectangular prism has 6 faces. Actually, the opposite faces are identical, so this figure has 3 pairs of faces. Also, a prism has only 3 dimensions: length, width, and height.



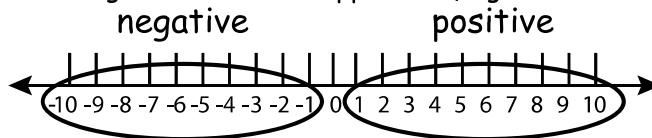
This prism has identical left & right sides (A & B), identical top and bottom (C & D), and identical front and back (unlabeled).

1. Find the area of the front:  $l \times w$ . ( $10 \text{ m} \times 5 \text{ m} = 50 \text{ m}^2$ )  
Since the back is identical, its area is the same.
2. Find the area of the top (C):  $l \times h$ . ( $10 \text{ m} \times 2 \text{ m} = 20 \text{ m}^2$ )  
Since the bottom (D) is identical, its area is the same.
3. Find the area of side A:  $w \times h$ . ( $2 \text{ m} \times 5 \text{ m} = 10 \text{ m}^2$ )  
Since side B is identical, its area is the same.
4. Add up the areas of all 6 faces.  
( $10 \text{ m}^2 + 10 \text{ m}^2 + 20 \text{ m}^2 + 20 \text{ m}^2 + 50 \text{ m}^2 + 50 \text{ m}^2 = 160 \text{ m}^2$ )

The formula is Surface Area =  $2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height})$   
or  $SA = 2lw + 2lh + 2wh$

## Ordering Integers

Integers include the counting numbers, their opposites (negative numbers) and zero.



The negative numbers are to the left of zero.

The positive numbers are to the right of zero.

When **ordering integers**, they are being arranged either from least to greatest or from greatest to least. The further a number is to the right, the greater its value. For example, 9 is further to the right than 2, so 9 is greater than 2.

In the same way, -1 is further to the right than -7, so -1 is greater than -7.

**Examples:** Order these integers from **least to greatest**. -10, 9, -25, 36, 0

Remember, the smallest number will be the one farthest to the left on the number line, -25, then -10, then 0. Next will be 9, and finally 36.

Answer: -25, -10, 0, 9, 36

Put these integers in order from **greatest to least**. -94, -6, -24, -70, -14  
Now the greatest value (farthest to the right) will come first and the smallest value (farthest to the left) will come last.

Answer: -6, -14, -24, -70, -94

# Help Pages

## Solved Examples

### Integers (continued)

The rules for performing operations (+, -, ×, ÷) on integers are very important and must be memorized.

#### The Addition Rules for Integers:

1. When the signs are the same, add the numbers and keep the sign.

$$\begin{array}{r} +33 \\ + +19 \\ \hline +52 \end{array} \qquad \begin{array}{r} -33 \\ + -19 \\ \hline -52 \end{array}$$

2. When the signs are different, subtract the numbers and use the sign of the larger number.

$$\begin{array}{r} +33 \\ + -19 \\ \hline +14 \end{array} \qquad \begin{array}{r} -55 \\ + +27 \\ \hline -28 \end{array}$$

#### The Subtraction Rule for Integers:

Change the sign of the second number and add (follow the Addition Rule for Integers above).

$$\begin{array}{r} +56 \\ - -26 \\ \hline \end{array} \xrightarrow{\text{apply rule}} \begin{array}{r} +56 \\ + +26 \\ \hline +82 \end{array} \qquad \begin{array}{r} +48 \\ - +23 \\ \hline \end{array} \xrightarrow{\text{apply rule}} \begin{array}{r} +48 \\ + -23 \\ \hline +25 \end{array}$$

Notice that every subtraction problem becomes an addition problem, using this rule!

#### The Multiplication and Division Rule for Integers:

1. When the signs are the same, the answer is positive (+).

$$+7 \times +3 = +21 \qquad -7 \times -3 = +21$$

$$+18 \div +6 = +3 \qquad -18 \div -6 = +3$$

2. When the signs are different, the answer is negative (-).

$$+7 \times -3 = -21 \qquad -7 \times +3 = -21$$

$$-18 \div +6 = -3 \qquad +18 \div -6 = -3$$

+		+		+
-	×	-		+
+		-		-
-		+		-
+		+	=	+
-	÷	-		+
+		-		-
-		+		-

The chart to the right contains a helpful summary of this rule.

### Proportion

A **proportion** is a statement that two ratios are equal to each other. There are two ways to solve a proportion when a number is missing.

1. One way to solve a proportion is already familiar to you. You can use the equivalent fraction method.

$$\begin{array}{c} \times 8 \\ \curvearrowright \\ \frac{5}{8} = \frac{n}{64} \\ \curvearrowleft \\ \times 8 \\ n = 40 \end{array}$$

So,  $\frac{5}{8} = \frac{40}{64}$ .

2. Another way to solve a proportion is by using cross-products.

To use Cross-Products:

1. Multiply downward on each diagonal.
2. Make the product of each diagonal equal to each other.
3. Solve for the missing variable.

$$\begin{array}{c} \frac{14}{20} = \frac{21}{n} \\ \swarrow \quad \searrow \\ 20 \times 21 = 14 \times n \\ 420 = 14n \\ \underline{420 = 14n} \\ 14 \quad \underline{14} \\ 30 = n \\ \text{So, } \frac{14}{20} = \frac{21}{30} \end{array}$$

## Help Pages

## Solved Examples

## Percent

When changing from a fraction to a percent, a decimal to a percent, or from a percent to either a fraction or a decimal, it is very helpful to use an FDP chart (Fraction, Decimal, Percent).

To change a **fraction to a percent and/or decimal**, first find an equivalent fraction with 100 in the denominator. Once you have found that equivalent fraction, it can easily be written as a decimal. To change that decimal to a percent, move the decimal point 2 places to the right and add a % sign.

**Example:** Change  $\frac{2}{5}$  to a percent and then to a decimal.

1. Find an equivalent fraction with 100 in the denominator.
2. From the equivalent fraction above, the decimal can easily be found. Say the name of the fraction: "forty hundredths." Write this as a decimal. 0.40
3. To change 0.40 to a percent, move the decimal two places to the right. Add a % sign. 40%

F	D	P
$\frac{2}{5}$		
F	D	P
$\frac{2}{5} = \frac{?}{100}$	0.40	
F	D	P
$\frac{2}{5} = \frac{?}{100}$	0.40	40%

$$\begin{array}{c} \times 20 \\ \curvearrowright \\ \frac{2}{5} = \frac{?}{100} \\ \curvearrowleft \\ \times 20 \end{array}$$

$$? = 40$$

$$\frac{2}{5} = \frac{40}{100} = 0.40$$

$$0.\underline{4}0 = 40\%$$

When changing from a **percent to a decimal or a fraction**, the process is similar to the one used on the page above. Begin with the percent. Write it as a fraction with a denominator of 100; reduce this fraction. Return to the percent, move the decimal point 2 places to the left. This is the decimal.

**Example:** Write 45% as a fraction and then as a decimal.

1. Begin with the percent. (45%) Write a fraction where the denominator is 100 and the numerator is the "percent."  $\frac{45}{100}$
2. This fraction must be reduced. The reduced fraction is  $\frac{9}{20}$ .
3. Go back to the percent. Move the decimal point two places to the left to change it to a decimal.

$$45\% = \frac{45}{100}$$

$$\frac{45(\div 5)}{100(\div 5)} = \frac{9}{20}$$

$$45\% = \underline{.45}$$

Decimal point goes here.

When changing from a **decimal to a percent or a fraction**, again, the process is similar to the one used above. Begin with the decimal. Move the decimal point 2 places to the right and add a % sign. Return to the decimal. Write it as a fraction and reduce.

**Example:** Write 0.12 as a percent and then as a fraction.

1. Begin with the decimal. (0.12) Move the decimal point two places to the right to change it to a percent.
2. Go back to the decimal and write it as a fraction. Reduce this fraction.

$$0.\underline{1}2 = 12\%$$

$$0.12 = \text{twelve hundredths}$$

$$\frac{12}{100} = \frac{12(\div 4)}{100(\div 4)} = \frac{3}{25}$$

## Help Pages

## Solved Examples

## Compound Probability

The probability of two or more independent events occurring together can be determined by multiplying the individual probabilities together. The product is called the compound probability.

Probability of A and B = (Probability of A)  $\times$  (Probability of B)

or  $P(A \text{ and } B) = P(A) \times P(B)$

**Example:** What is the probability of rolling a 6 and then a 2 on two rolls of a die [  $P(6 \text{ and } 2)$  ]?

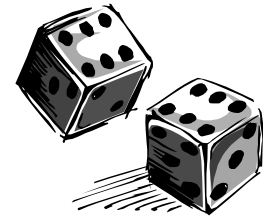
A) First, find the probability of rolling a 6 [  $P(6)$  ]. Since there are 6 numbers on a die and only one of them is a 6, the probability of getting a 6 is  $\frac{1}{6}$ .

B) Then find the probability of rolling a 2 [  $P(2)$  ].

Since there are 6 numbers on a die and only one of them is a 2, the probability of getting a 2 is  $\frac{1}{6}$ .

So,  $P(6 \text{ and } 2) = P(6) \times P(2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

There is a 1 to 36 chance of getting a 6 and then a 2 on two rolls of a die.



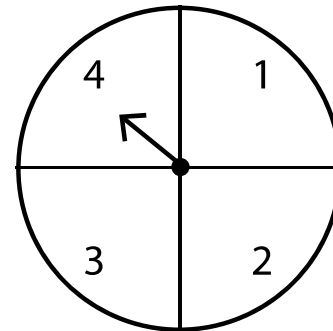
**Example:** What is the probability of getting a 4 and then a number greater than 2 on two spins of this spinner [  $P(4 \text{ and greater than } 2)$  ]?

A) First, find the probability of getting a 4 [  $P(4)$  ]. Since there are 4 numbers on the spinner and only one of them is a 4, the probability of getting a 4 is  $\frac{1}{4}$ .

B) Then find the probability of getting a number greater than 2 [  $P(\text{greater than } 2)$  ]. Since there are 4 numbers on the spinner and two of them are greater than 2, the probability of getting a 2 is  $\frac{2}{4}$ .

So,  $P(2 \text{ and greater than } 2) = P(2) \times P(\text{greater than } 2) = \frac{1}{4} \times \frac{2}{4} = \frac{2}{16} = \frac{1}{8}$

There is a 1 to 8 chance of getting a 4 and then a number greater than 2 on two spins of a spinner.



**Example:** On three flips of a coin, what is the probability of getting heads, tails, heads [  $P(H,T,H)$  ]?

A) First, find the probability of getting heads [  $P(H)$  ].

Since there are only 2 sides on a coin and only one of them is heads, the probability of getting heads is  $\frac{1}{2}$ .

B) Then find the probability of getting tails [  $P(T)$  ]. Again, there are only 2 sides on a coin and only one of them is tails. The probability of getting tails is also  $\frac{1}{2}$ .

So,  $P(H,T,H) = P(H) \times P(T) \times P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

There is a 1 to 8 chance of getting heads, tails and then heads on 3 flips of a coin.



## Who Knows???

Degrees in a right angle? .....(90)	Number with only 2 factors? .... (prime)
A straight angle? ..... (180)	Perimeter? .....(add the sides)
Angle greater than 90°? ..... (obtuse)	Area? .....(length x width)
Less than 90°? .....(acute)	Volume? ..... (length x width x height)
Sides in a quadrilateral? .....(4)	Area of parallelogram?..... ..... (base x height)
Sides in an octagon?..... (8)	Area of triangle? .....( $\frac{1}{2}$ base x height)
Sides in a hexagon? .....(6)	Area of trapezoid? ..... .....( $\frac{\text{base} + \text{base}}{2} \times \text{height}$ )
Sides in a pentagon? ..... (5)	Surface area of a rectangular prism? ..... $2(lw) + 2(wh) + 2(lh)$
Sides in a heptagon? ..... (7)	Area of a circle? ..... ( $\pi r^2$ )
Sides in a nonagon? ..... (9)	Circumference of a circle? ..... ( $\pi d$ )
Sides in a decagon? ..... (10)	Triangle with no sides equal? ..... ..... (scalene)
Inches in a yard? ..... (36)	Triangle with 3 sides equal?..... ..... (equilateral)
Yards in a mile? .....(1,760)	Triangle with 2 sides equal? ..... ..... (isosceles)
Feet in a mile? ..... (5,280)	Distance across the middle of a circle? ..... (diameter)
Centimeters in a meter? ..... (100)	Half of the diameter? ..... (radius)
Teaspoons in a tablespoon? ..... (3)	Figures with the same size and shape? ..... (congruent)
Ounces in a pound? .....(16)	Figures with same shape, different sizes? .....(similar)
Pounds in a ton? .....(2,000)	Number occurring most often? ..... ..... (mode)
Cups in a pint? ..... (2)	Middle number? ..... (median)
Pints in a quart? ..... (2)	Answer in addition? .....(sum)
Quarts in a gallon? .....(4)	Answer in division? .....(quotient)
Millimeters in a meter? ..... (1,000)	Answer in multiplication? ..... (product)
Years in a century? ..... (100)	Answer in subtraction? .....(difference)
Years in a decade? .....(10)	
Celsius freezing? ..... (0°C)	
Celsius boiling? .....(100°C)	
Fahrenheit freezing? .....(32°F)	
Fahrenheit boiling? .....(212°F)	