Simple Solutions.

Minutes a Day–Mastery for a Lifetime!

Intermediate B

Mathematics

Help Pages & "Who Knows?"

Help Pages

Vocabulary

Arithmetic Operations

difference — the result or answer to a subtraction problem. **Example**: The difference of 5 and 1 is 4.

product — the result or answer to a multiplication problem. **Example**: The product of 5 and 3 is 15.

quotient — the result or answer to a division problem. **Example**: The quotient of 8 and 2 is 4.

sum — the result or answer to an addition problem. **Example**: The sum of 5 and 2 is 7.

Factors and Multiples

factors — are multiplied together to get a product. **Example**: 2 and 3 are factors of 6.

multiples — can be evenly divided by a number. **Example**: 5, 10, 15 and 20 are multiples of 5.

composite number — a number with more than 2 factors.
Example: 10 has factors of 1, 2, 5, and 10. Ten is a composite number.

prime number — a number with exactly 2 factors (the number itself and 1). 1 is not prime (it has only 1 factor). **Example**: 7 has factors of 1 and 7. Seven is a prime number.

greatest common factor (GCF) — the highest factor that 2 numbers have in common.
Example: The factors of 6 are 1, 2, 3, and 6. The factors of 9 are 1, 3, and 9. The GCF of 6 and 9 is 3.

least common multiple (LCM) — the smallest multiple that 2 numbers have in common.
Example: Multiples of 3 are 3, 6, 9, 12, 15… Multiples of 4 are 4, 8, 12, 16… The LCM of 3 and 4 is 12.

prime factorization — a number, written as a product of its prime factors.
Example: 140 can be written as 2 x 2 x 5 x 7 or 2^2 x 5 x 7. (All of these are prime factors of 140.)

Fractions and Decimals

improper fraction — a fraction in which the numerator is larger than the denominator. **Example**: $\frac{9}{4}$

mixed number — the sum of a whole number and a fraction. **Example**: $5\frac{1}{4}$

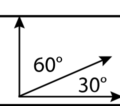
reciprocal — a fraction where the numerator and denominator are interchanged. The product of a fraction and its reciprocal is always 1.
Example: The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. $\frac{3}{5} \times \frac{5}{3} = \frac{15}{15} = 1$

repeating decimal — a decimal in which a number or a series of numbers continues on and on.
Example: 2.33333333, 4.151515151515, 7.125555555, etc.

Geometry

acute angle — an angle measuring less than 90°.

complementary angles — two angles whose measures add up to 90°.



congruent — figures with the same shape and the same size.

obtuse angle — an angle measuring more than 90°.

right angle — an angle measuring exactly 90°.

similar — figures having the same shape, but different size.

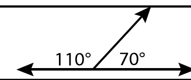
Help Pages

Vocabulary (continued)

Geometry

straight angle — an angle measuring exactly 180°.

supplementary angles — two angles whose measures add up to 180°.



surface area — the sum of the areas of all of the faces of a solid figure.

Geometry — Circles

circumference — the distance around the outside of a circle.

diameter — the widest distance across a circle. The diameter always passes through the center.

radius — the distance from any point on the circle to the center. The radius is half of the diameter.

Geometry — Polygons

Number of Sides	Name	Number of Sides	Name
3	triangle	7	heptagon
4	quadrilateral	8	octagon
5	pentagon	9	nonagon
6	hexagon	10	decagon

Geometry — Triangles

equilateral — a triangle in which all 3 sides have the same length.

isosceles — a triangle in which 2 sides have the same length.

scalene — a triangle in which no sides are the same length.

Measurement — Relationships

Volume	Distance
3 teaspoons in a tablespoon	36 inches in a yard
2 cups in a pint	1,760 yards in a mile
2 pints in a quart	5,280 feet in a mile
4 quarts in a gallon	100 centimeters in a meter
Weight	1,000 millimeters in a meter
16 ounces in a pound	**Temperature**
2,000 pounds in a ton	0°Celsius – Freezing Point
Time	100°Celsius – Boiling Point
10 years in a decade	32°Fahrenheit – Freezing Point
100 years in a century	212°Fahrenheit – Boiling Point

Help Pages

Vocabulary (continued)

Ratio and Proportion
proportion — a statement that two ratios (or fractions) are equal. **Example**: $\frac{1}{2}=\frac{3}{6}$
ratio — a comparison of two numbers by division; a ratio looks like a fraction. **Example**: $\frac{2}{5}$ or 2 to 5 or 2 : 5
percent (%) —the ratio of any number to 100. **Example**: 14% means 14 out of 100 or $\frac{14}{100}$.

Statistics
mean — the average of a group of numbers. The mean is found by finding the sum of a group of numbers and then dividing the sum by the number of members in the group. **Example**: The average of 12, 18, 26, 17, and 22 is **19**. $\frac{12+18+26+17+22}{5}=\frac{95}{5}=19$
median — the middle value in a group of numbers. The median is found by listing the numbers in order from least to greatest, and finding the one that is in the middle of the list. If there is an even number of members in the group, the median is the average of the two middle numbers. **Example**: The median of 14, 17, 24, 11, and 26 is **17**. 11, 14, (17), 24, 26 The median of 77, 93, 85, 95, 70, and 81 is **83**. 70, 77, (81, 85), 93, 95 $\frac{81+85}{2}=83$
mode — the number that occurs most often in a group of numbers. The mode is found by counting how many times each number occurs in the list. The number that occurs more than any other is the mode. Some groups of numbers have more than one mode. **Example**: The mode of 77, (93), 85, (93), 77, 81, (93), and 71 is **93**. (93 occurs more than the others.)

Place Value

Whole Numbers

8,	9	6	3,	2	7	1,	4	0	5
Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

The number above is read: eight billion, nine hundred sixty-three million, two hundred seventy-one thousand, four hundred five.

Help Pages

Place Value

Decimal Numbers

1	7	8	.	6	4	0	5	9	2
Hundreds	Tens	Ones	*Decimal Point*	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths

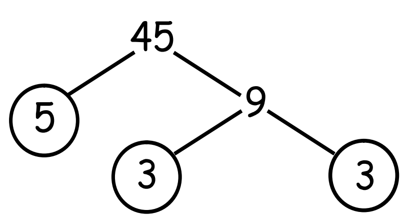
The number above is read: one hundred seventy-eight and six hundred forty thousand, five hundred ninety-two millionths.

Solved Examples

Factors and Multiples

The **prime factorization** of a number is when a number is written as a product of its prime factors. A factor tree is helpful in finding the prime factors of a number.

Example: Use a factor tree to find the prime factors of 45.



1. Find any 2 factors of 45 (5 and 9).
2. If a factor is prime, circle it. If a factor is not prime, find 2 factors of it.
3. Continue until all factors are prime.
4. In the final answer, the prime factors are listed in order, least to greatest, using exponents when needed.

The prime factorization of 45 is $3 \times 3 \times 5$ or $3^2 \times 5$.

The **greatest common factor (GCF)** is the largest factor that 2 numbers have in common.

Example: Find the greatest common factor (GCF) of 32 and 40.

The factors of 32 are 1, 2, 4, (8), 16, 32

The factors of 40 are 1, 2, 4, 5, (8), 10, 20, 40

The GCF of 32 and 40 is **8**.

1. First list the factors of each number.
2. Find the largest number that is in both lists.

Help Pages

Solved Examples

Factors and Multiples

The **least common multiple (LCM)** is the smallest multiple that two numbers have in common. The prime factors of the numbers can be useful in finding the LCM.

Example: Find the least common multiple (LCM) of 16 and 24.

```
2 | 16, 24
  2 | 8, 12
    2 | 4, 6
      2 | 2, 3
        3 | 1, 3
            1, 1
```

1. If any of the numbers are even, factor out a 2.
2. Continue factoring out 2 until all numbers left are odd.
3. If the prime number cannot be divided evenly into the number, simply bring the number down.
4. Once you are left with all 1's at the bottom, you're finished!
5. Multiply all of the prime numbers (on the left side of the bracket) together to find the least common multiple.

The LCM of 16 and 24 is
2 X 2 X 2 X 2 X 3 or **48**.

Fractions

Changing from an **improper fraction** to a **mixed number.**

Example: Change the improper fraction $\frac{5}{2}$ to a mixed number.

$\frac{5}{2}$ (five halves) means $5 \div 2$.

So, $\frac{5}{2}$ is equal to 2 wholes and 1 half or $2\frac{1}{2}$.

```
   2 wholes
2)5
 -4
  1 half
```

Changing from a mixed number to an improper fraction.

Example: Change the mixed number $7\frac{1}{4}$ to an improper fraction.

1. You're going to make a new fraction. To find the numerator of the new fraction, multiply the whole number by the denominator, and add the numerator
2. Keep the same denominator in your new fraction as you had in the mixed number.

$7\frac{1}{4}$ 7 x 4 = 28. 28 + 1 = 29.

The new numerator is 29.

Keep the same denominator, 4.

The new fraction is $\frac{29}{4}$.

$7\frac{1}{4}$ is equal to $\frac{29}{4}$.

Help Pages

Solved Examples

Fractions (continued)

Equivalent fractions are two fractions that are equal to each other. Usually you will be finding a missing numerator or denominator.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a denominator of 35.

$$\frac{4}{5} \overset{\times 7}{=} \frac{?}{35}$$

1. Ask yourself, "What did I do to 5 to get 35?" (Multiply by 7.)
2. Whatever you did in the denominator, you also must do in the numerator. 4 x 7 = 28. The missing numerator is 28.

So, $\frac{4}{5}$ is equivalent to $\frac{28}{35}$.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a numerator of 24.

$$\frac{4}{5} \overset{\times 6}{=} \frac{24}{?}$$

1. Ask yourself, "What did I do to 4 to get 24?" (Multiply by 6.)
2. Whatever you did in the numerator, you also must do in the denominator. 5 x 6 = 30. The missing denominator is 30.

So, $\frac{4}{5}$ is equivalent to $\frac{24}{30}$.

Comparing fractions means looking at two or more fractions and determining if they are equal, if one is greater than (>) the other, or if one is less than (<) the other. A simple way to compare fractions is by cross-multiplying, using the steps below.

Examples: Compare these fractions. Use the correct symbol. $\frac{8}{9} \bigcirc \frac{3}{4}$ $\frac{7}{9} \bigcirc \frac{6}{7}$

$32 > 27$: $\frac{8}{9} \otimes \frac{3}{4}$

$49 < 54$: $\frac{7}{9} \otimes \frac{6}{7}$

1. Begin with the denominator on the left and multiply by the opposite numerator. Put the answer (product) above the side where you ended. (9 x 3 = 27)
2. Cross-multiply the other denominator and numerator and put that product above where you ended.
3. Compare the two products and insert the correct symbol.

   HINT: **Always** multiply diagonally **upwards**!

So, $\frac{8}{9} > \frac{3}{4}$ and $\frac{7}{9} < \frac{6}{7}$

To **add (or subtract) fractions with the same denominator**, simply add (or subtract) the numerators, keeping the same denominator.

Examples:

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

$$\frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

To **add mixed numbers**, follow a process similar to the one you used with fractions. If the sum is an improper fraction, be sure to simplify it.

Example:

$$\begin{array}{r} 1\frac{2}{5} \\ +1\frac{4}{5} \\ \hline 2\frac{6}{5} \end{array}$$

$2\frac{6}{5}$ is improper. $\frac{6}{5}$ can be rewritten as $1\frac{1}{5}$.

So, $2\frac{6}{5}$ is $2 + 1\frac{1}{5} = 3\frac{1}{5}$.

Help Pages

Solved Examples

Fractions (continued)

When **adding fractions that have different denominators**, you need to change the fractions so they have a common denominator before they can be added.

Finding the **least common denominator (LCD)**:

The LCD of the fractions is the same as the least common multiple of the denominators. Sometimes, the LCD will be the product of the denominators.

Example: Find the sum of $\frac{3}{8}$ and $\frac{1}{12}$.

$$\begin{array}{r} \frac{3}{8} = \frac{9}{24} \\ +\frac{1}{12} = \frac{2}{24} \\ \hline \frac{11}{24} \end{array}$$

1. First, find the LCM of 8 and 12.
2. The LCM of 8 and 12 is 24. This is also the LCD of these 2 fractions.
3. Find an equivalent fraction for each that has a denominator of 24.
4. When they have a common denominator, the fractions can be added.

$$\begin{array}{r|l} 2 & 8, 12 \\ 2 & 4, 6 \\ 2 & 2, 3 \\ 3 & 1, 3 \\ & 1, 1 \end{array} \qquad 2 \times 2 \times 2 \times 3 = 24$$

The LCM is 24.
So, the LCD is 24.

Example: Add $\frac{1}{4}$ and $\frac{1}{5}$.

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ +\frac{1}{5} = \frac{4}{20} \\ \hline \frac{9}{20} \end{array}$$

$4 \times 5 = 20$ The LCD is 20.

When **adding mixed numbers with unlike denominators**, follow a process similar to the one you used with fractions (above). Be sure to put your answer in simplest form.

Example: Find the sum of $6\frac{3}{7}$ and $5\frac{2}{3}$.

$$\begin{array}{r} 6\frac{3}{7} = 6\frac{9}{21} \\ +5\frac{2}{3} = 5\frac{14}{21} \\ \hline 11\frac{23}{21} \end{array}$$

improper

$\frac{23}{21} = 1\frac{2}{21} + 11 = 12\frac{2}{21}$

1. Find the LCD.
2. Find the missing numerators.
3. Add the whole numbers, then add the fractions.
4. Make sure your answer is in simplest form.

Help Pages

Solved Examples

Fractions (continued)

When **subtracting numbers with unlike denominators**, follow a process similar to the one you used when adding fractions. Be sure to put your answer in simplest form.

Examples: Find the difference of $\frac{3}{4}$ and $\frac{2}{5}$.

$$\frac{3}{4} = \frac{15}{20}$$
$$-\frac{2}{5} = \frac{8}{20}$$
$$\frac{7}{20}$$

1. Find the LCD just as you did when adding fractions.
2. Find the missing numerators.
3. Subtract the numerators and keep the common denominator.
4. Make sure your answer is in simplest form.

Subtract $\frac{1}{16}$ from $\frac{3}{8}$.

$$\frac{3}{8} = \frac{6}{16}$$
$$-\frac{1}{16} = \frac{1}{16}$$
$$\frac{5}{16}$$

When **subtracting mixed numbers with unlike denominators**, follow a process similar to the one you used when adding mixed numbers. Be sure to put your answer in simplest form.

Example: Subtract $4\frac{2}{5}$ from $8\frac{9}{10}$.

1. Find the LCD.
2. Find the missing numerators.
3. Subtract and simplify your answer.

$$8\frac{9}{10} = 8\frac{9}{10}$$
$$-4\frac{2}{5} = 4\frac{4}{10}$$
$$4\frac{5}{10} = 4\frac{1}{2}$$

Sometimes when subtracting mixed numbers you may need to rename them. If the numerator of the top fraction is smaller than the numerator of the bottom fraction, rename the mixed number as in the example below.

Example: Subtract $5\frac{5}{6}$ from $9\frac{1}{4}$.

1. Find the LCD.
2. Find the missing numerators. Write equivalent fractions.
3. Because you can't subtract 10 from 3, rename the whole number as a mixed number using the common denominator.
4. Add the two fractions to get an improper fraction.
5. Subtract the whole numbers and the fractions and simplify your answer.

$$9\frac{1}{4} = 9\frac{3}{12} = 8\frac{12}{12} + \frac{3}{12} = 8\frac{15}{12}$$
$$-5\frac{5}{6} = 5\frac{10}{12} = \qquad 5\frac{10}{12}$$
$$3\frac{5}{12}$$

More examples:

$$8\frac{1}{2} = 8\frac{2}{4} = 7\frac{4}{4} + \frac{2}{4} = 7\frac{6}{4}$$
$$-4\frac{3}{4} = 4\frac{3}{4} = \qquad -4\frac{3}{4}$$
$$3\frac{3}{4}$$

$$10\frac{1}{5} = 10\frac{4}{20} = 9\frac{20}{20} + \frac{4}{20} = 9\frac{24}{20}$$
$$-6\frac{3}{4} = 6\frac{15}{20} = \qquad -6\frac{15}{20}$$
$$3\frac{9}{20}$$

Help Pages

Solved Examples

Fractions (continued)

To **multiply fractions**, simply multiply the numerators together to get the numerator of the product. Then multiply the denominators together to get the denominator of the product. Make sure your answer is in simplest form.

1. Multiply the numerators.
2. Multiply the denominators.
3. Simplify your answer.

Examples: Multiply $\frac{3}{5}$ by $\frac{2}{3}$.

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$$

Multiply $\frac{5}{8}$ by $\frac{4}{5}$.

$$\frac{5}{8} \times \frac{4}{5} = \frac{20}{40} = \frac{1}{2}$$

Sometimes you can use cancelling when multiplying fractions. Let's look at the examples again.

1. Are there any numbers in the numerator and the denominator that have common factors?
2. If so, cross out the numbers, divide both by that factor, and write the quotient.
3. Then, multiply the fractions as described above, using the quotients instead of the original numbers.

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{2}{\underset{1}{\cancel{3}}} = \frac{2}{5}$$

The 3s have a common factor — 3. Divide both of them by 3. Since, 3 ÷ 3 = 1, we cross out the 3s and write 1s in their place.

Now, multiply the fractions. In the numerator, 1 × 2 = 2. In the denominator, 5 × 1 = 5 .

The answer is $\frac{2}{5}$.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}} = \frac{1}{2}$$

As in the other example, the 5s can be cancelled.

But here, the 4 and the 8 also have a common factor — 4.

$8 \div 4 = 2$ and $4 \div 4 = 1$.

After cancelling both of these, you can multiply the fractions.

REMEMBER: You can cancel up and down or diagonally, but NEVER sideways!

When **multiplying mixed numbers**, you must first change them into improper fractions.

1. Change each mixed number to an improper fraction.
2. Cancel wherever you can.
3. Multiply the fractions.
4. Put your answer in simplest form.

Examples: Multiply $2\frac{1}{4}$ by $3\frac{1}{9}$.

$$2\frac{1}{4} \times 3\frac{1}{9} =$$

$$\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{7}{\cancel{28}}}{\underset{1}{\cancel{9}}} = \frac{7}{1} = 7$$

Multiply $3\frac{1}{8}$ by 4.

$$3\frac{1}{8} \times 4 =$$

$$\frac{25}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{1} = \frac{25}{2} = 12\frac{1}{2}$$

To **divide fractions**, you must take the reciprocal of the 2nd fraction, and then multiply that reciprocal by the 1st fraction. Don't forget to simplify your answer!

1. Keep the 1st fraction as it is.
2. Write the reciprocal of the 2nd fraction.
3. Change the sign to multiplication.
4. Cancel if you can and multiply.
5. Simplify your answer.

Examples: Divide $\frac{1}{2}$ by $\frac{7}{12}$.

$$\frac{1}{2} \div \frac{7}{12} =$$

$$\frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{6}{\cancel{12}}}{7} = \frac{6}{7}$$

Divide $\frac{7}{8}$ by $\frac{3}{4}$.

$$\frac{7}{8} \div \frac{3}{4} =$$

$$\frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{3} = \frac{7}{6} = 1\frac{1}{6}$$

Help Pages

Solved Examples

Fractions (continued)

When **dividing mixed numbers**, you must first change them into improper fractions.

Example: Divide $1\frac{1}{4}$ by $3\frac{1}{2}$.

$$1\frac{1}{4} \div 3\frac{1}{2} =$$
$$\frac{5}{4} \div \frac{7}{2} =$$
$$\frac{5}{\cancel{4}_2} \times \frac{\cancel{2}^1}{7} = \frac{5}{14}$$

1. Change each mixed number to an improper fraction.
2. Keep the 1st fraction as it is.
3. Write the reciprocal of the 2nd fraction.
4. Change the sign to multiplication.
5. Cancel if you can and multiply.
6. Simplify your answer.

Decimals

When we **compare decimals**, we are looking at two or more decimal numbers and deciding which has the smaller or larger value. We sometimes compare by placing them in order from least to greatest or from greatest to least. Another way to compare is to use the symbols for "less than" (<), "greater than" (>), or "equal to" (=).

Example: Order these numbers from least to greatest. 0.561 0.506 0.165

1. Write the numbers in a column, lining up the decimal points.
2. Write zeroes, if necessary, so all have the same number of digits.
3. Begin on the left and compare the digits.

0.561
0.506
0.165

Since they all have 3 digits, we don't need to add zeroes.

Beginning on the left, the fives are equal, but the one is less, so 0.165 is the smallest.

Then, look at the next digit. The zero is less than the six, so 0.506 is next smallest.

So, in order from least to greatest:

0.165, 0.506, 0.561

Example: Place these numbers in order from greatest to least. 0.44 0.463 0.045

0.440
0.463
0.045

After lining up the numbers, we must add a zero to 0.44 to make them all have the same number of digits.

Beginning on the left, the zero is smaller than the fours, so 0.045 is the smallest.

Look at the next digit. The four is smaller than the six, so 0.440 is the next smallest.

In order from greatest to least: 0.463, 0.440, 0.045

Help Pages

Solved Examples

Decimals (continued)

When we **round decimals**, we are approximating them. This means we end the decimal at a certain place value and we decide if it's closer to the next higher number (round up) or to the next lower number (keep the same). It might be helpful to look at the decimal place-value chart on p. 287.

Example: Round 0.574 to the tenths place.

There is a 5 in the rounding (tenths) place. → 0.574

Since 7 is greater than 5, change the 5 to a 6. → 0.574

Drop the digits to the right of the tenths place. → 0.6

1. Identify the number in the rounding place.
2. Look at the digit to its right.
3. If the digit is 5 or greater, increase the number in the rounding place by 1. If the digit is less than 5, keep the number in the rounding place the same.
4. Drop all digits to the right of the rounding place.

Example: Round 2.783 to the nearest hundredth.

2.783 ← There is an 8 in the rounding place.

2.783 ← Since 3 is less than 5, keep the rounding place the same.

2.78 ← Drop the digits to the right of the hundredths place.

Adding and subtracting decimals is very similar to adding or subtracting whole numbers. The main difference is that you have to line-up the decimal points in the numbers before you begin.

Examples: Find the sum of 3.14 and 1.2.

```
  3.14
+ 1.20
  4.34
```

Add 55.1, 6.472, and 18.33.

```
 55.100
  6.472
+18.330
 79.902
```

1. Line up the decimal points. Add zeroes as needed.
2. Add (or subtract) the decimals.
3. Add (or subtract) the whole numbers.
4. Bring the decimal point straight down.

Examples: Subtract 3.7 from 9.3.

```
  9.3
- 3.7
  5.6
```

Find the difference of 4.1 and 2.88.

```
  4.10
- 2.88
  1.22
```

Help Pages

Solved Examples

Decimals (continued)

When **multiplying a decimal by a whole number**, the process is similar to multiplying whole numbers.

Examples: Multiply 3.42 by 4.

$$\begin{array}{r} 3.42 \\ \times\ 4 \\ \hline 13.68 \end{array}$$

- 3.42 → 2 decimal places
- 4 → 0 decimal places
- 13.68 → Place decimal point so there are 2 decimal places.

1. Line up the numbers on the right.
2. Multiply. Ignore the decimal point.
3. Place the decimal point in the product. (The total number of decimal places in the product must equal the total number of decimal places in the factors.)

Find the product of 2.3 and 2.

$$\begin{array}{r} 2.3 \\ \times\ 2 \\ \hline 4.6 \end{array}$$

- 2.3 → 1 decimal place
- 2 → 0 decimal places
- 4.6 → Place decimal point so there is 1 decimal place.

The process for **multiplying two decimal numbers** is a lot like what we just did above.

Examples: Multiply 0.4 by 0.6.

$$\begin{array}{r} 0.4 \\ \times\ 0.6 \\ \hline 0.24 \end{array}$$

- 0.4 → 1 decimal place
- 0.6 → 1 decimal place
- 0.24 → Place decimal point so there are 2 decimal places.

Find the product of 2.67 and 0.3.

$$\begin{array}{r} 2.67 \\ \times\ 0.3 \\ \hline 0.801 \end{array}$$

- 2.67 → 2 decimal places
- 0.3 → 1 decimal place
- 0.801 → Place decimal point so there are 3 decimal places.

Sometimes it is necessary to add **zeroes in the product** as placeholders in order to have the correct number of decimal places.

Example: Multiply 0.03 by 0.4.

$$\begin{array}{r} 0.03 \\ \times\ 0.4 \\ \hline 0.012 \end{array}$$

- 0.03 → 2 decimal places
- 0.4 → 1 decimal place
- 0.012 → Place decimal point so there are 3 decimal places.

We had to add a zero in front of the 12 so that we could have 3 decimal places in the product.

The process for **dividing a decimal number by a whole number** is similar to dividing whole numbers.

Examples: Divide 6.4 by 8.

$$\begin{array}{r} 0.8 \\ 8\overline{)6.4} \\ -6\ 4 \\ \hline 0 \end{array}$$

1. Set up the problem for long division.
2. Place the decimal point in the quotient directly above the decimal point in the dividend.
3. Divide. Add zeroes as placeholders if necessary. (See examples below.)

Find the quotient of 20.7 and 3.

$$\begin{array}{r} 6.9 \\ 3\overline{)20.7} \\ -18 \\ \hline 27 \\ -27 \\ \hline 0 \end{array}$$

Examples: Divide 4.5 by 6.

$$\begin{array}{r} 0.75 \\ 6\overline{)4.50} \\ -42\downarrow \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$

Add zero(es).

Bring zero down.

Keep dividing.

Find the quotient of 3.5 and 4.

$$\begin{array}{r} 0.875 \\ 4\overline{)3.500} \\ -3\ 2\downarrow\downarrow \\ \hline 30\downarrow \\ -28\downarrow \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

Help Pages

Solved Examples

Decimals (continued)

When dividing decimals the remainder is not always zero. Sometimes, the division continues on and on and the remainder begins to repeat itself. When this happens the quotient is called a **repeating decimal**.

Examples: Divide 2 by 3.

$$
\begin{array}{r}
0.\overline{66} \\
3\,\overline{)\,2.000} \\
-\;1\,8\downarrow\downarrow \\
\hline
2\,0\downarrow \\
-\;1\,8\downarrow \\
\hline
2\,0
\end{array}
$$

Divide 10 by 11.

$$
\begin{array}{r}
0.\overline{9090} \\
11\,\overline{)\,10.00000} \\
-\;99\downarrow\downarrow\downarrow\downarrow \\
\hline
100\downarrow\downarrow \\
-\;99\downarrow\downarrow \\
\hline
100
\end{array}
$$

Add zeroes as needed

This pattern (with the same remainder begins to repeat itself.

To write the final answer, put a bar in the quotient over the digits that repeat.

The process for **dividing a decimal number by a decimal number** is similar to other long division that you have done. The main difference is that we have to move the decimal point in both the dividend and the divisor the <u>same number of places</u> to the right.

1. Change the divisor to a whole number by moving the decimal point as many places to the right as needed.
2. Move the decimal in the dividend the same number of places to the right as you did in the divisor.
3. Put the decimal point in the quotient directly above the decimal point in the dividend.
4. Divide.

Examples: Divide 1.8 by 0.3.

$$
\begin{array}{r}
6. \\
0.3\,\overline{)\,1.8} \\
-18 \\
\hline
0
\end{array}
$$

Divide 0.385 by 0.05.

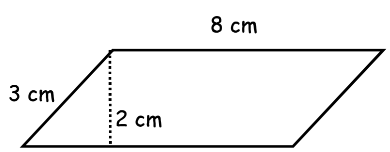
$$
\begin{array}{r}
7.7 \\
0.05\,\overline{)\,0.385} \\
-35\downarrow \\
\hline
35 \\
-35 \\
\hline
0
\end{array}
$$

Geometry

Finding the **area of a parallelogram** is similar to finding the area of any other quadrilateral. The area of the figure is equal to the length of its base multiplied by the height of the figure.

Area of parallelogram = base × height or $A = b \times h$

Example: Find the area of the parallelogram below.



1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down – never slanted.)
3. Multiply to find the area. (16 cm^2)

So, $A = 8 \text{ cm} \times 2 \text{ cm} = 16 \text{ cm}^2$.

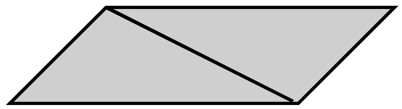
Help Pages

Solved Examples

Geometry (continued)

To find the **area of a triangle**, it is helpful to recognize that any triangle is exactly half of a parallelogram.

The whole figure is a parallelogram.

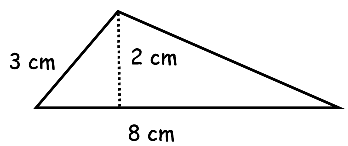


Half of the whole figure is a triangle.

So, the triangle's area is equal to half of the product of the base and the height.

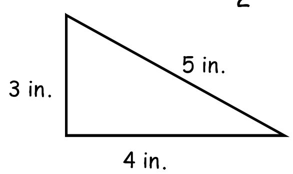
Area of triangle = $\frac{1}{2}$(base × height) or $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Examples: Find the area of the triangles below.



1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down – never slanted.)
3. Multiply them together and divide by 2 to find the area. (8 cm^2)

So, $A = 8\text{ cm} \times 2\text{ cm} \times \frac{1}{2} = 8\text{ cm}^2$.

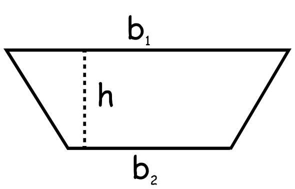


The base of this triangle is 4 inches long. Its height is 3 inches. (Remember the height is always straight up and down!)

So, $A = 4\text{ in.} \times 3\text{ in.} \times \frac{1}{2} = 6\text{ in}^2$.

Finding the **area of a trapezoid** is a little different than the other quadrilaterals that we have seen. Trapezoids have 2 bases of unequal length. To find the area, first find the average of the lengths of the 2 bases. Then, multiply that average by the height.

Area of trapezoid = $\frac{\text{base}_1 + \text{base}_2}{2} \times \text{height}$ or $A = \left(\frac{b_1 + b_2}{2}\right) \times h$

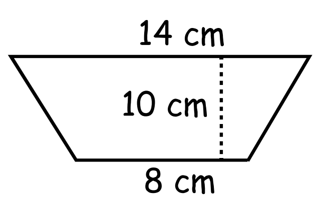


The bases are labeled b_1 and b_2.

The height, h, is the distance between the bases.

Example: Find the area of the trapezoid below.

1. Add the lengths of the two bases. (22 cm)
2. Divide the sum by 2. (11 cm)
3. Multiply that result by the height to find the area. (110 cm^2)



$\frac{14\text{ cm} + 8\text{ cm}}{2} = \frac{22\text{ cm}}{2} = 11\text{ cm}$

11 cm × 10 cm = **110 cm^2** = Area

Help Pages

Solved Examples

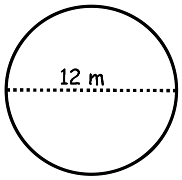
Geometry (continued)

The **circumference of a circle** is the distance around the outside of the circle. Before you can find the circumference of a circle you must know either its radius or its diameter. Also, you must know the value of the constant, pi (π) . $\pi = 3.14$ (rounded to the nearest hundredth)

Once you have this information, the circumference can be found by multiplying the diameter by pi.

Circumference = $\pi \times$ diameter or $C = \pi d$

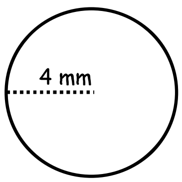
Examples: Find the circumference of the circles below.



1. Find the length of the diameter. (12 m)
2. Multiply the diameter by π . (12 m $\times$ 3.14)
3. The product is the circumference. (37.68 m)

So, C = 12 m $\times$ 3.14 = 37.68 m.

Sometimes the radius of a circle is given instead of the diameter. Remember, the radius of any circle is exactly half of the diameter. If a circle has a radius of 3 feet, its diameter is 6 feet.



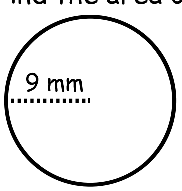
1. Since the radius is 4 mm, the diameter must be 8 mm.
2. Multiply the diameter by π . (8 mm $\times$ 3.14)
3. The product is the circumference. (25.12 mm)

So, C = 8 mm $\times$ 3.14 = 25.12 mm.

When finding the **area of a circle**, the length of the radius is squared (multiplied by itself), and then that answer is multiplied by the constant, pi (π). π = 3.14 (rounded to the nearest hundredth).

Area = π × radius × radius or $A = \pi r^2$

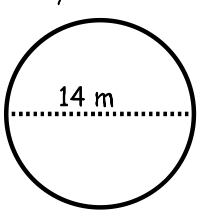
Examples: Find the area of the circles below.



So, A = 9 mm x 9 mm x 3.14 = **254.34 mm²**.

1. Find the length of the radius. (9 mm)
2. Multiply the radius by itself. (9 mm x 9 mm)
3. Multiply the product by pi. (81 mm^2 x 3.14)
4. The result is the area. (254.34 mm^2)

Sometimes the diameter of a circle is given instead of the radius. Remember, the diameter of any circle is exactly twice the radius. If a circle has a diameter of 6 feet, its radius is 3 feet.



1. Since the diameter is 14 m, the radius must be 7 m.
2. Square the radius. (7 m x 7 m)
3. Multiply that result by π . (49 m^2 × 3.14).
4. The product is the area. (153.86 m^2)

So, $A = (7\ m)^2 \times 3.14 =$ **153.86 m^2**.

Help Pages

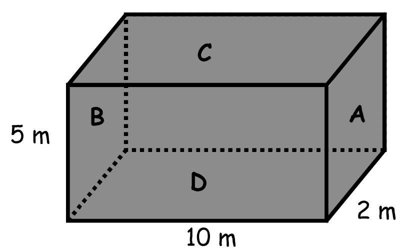
Solved Examples

Geometry (continued)

To find the **surface area** of a solid figure, it is necessary to first count the total number of faces. Then, find the area of each of the faces; finally, add the areas of each face. That sum is the surface area of the figure.

Here, the focus will be on finding the surface area of a rectangular prism. A rectangular prism has 6 faces. Actually, the opposite faces are identical, so this figure has 3 pairs of faces. Also, a prism has only 3 dimensions: length, width, and height.

This prism has identical left & right sides (A & B), identical top and bottom (C & D), and identical front and back (unlabeled).

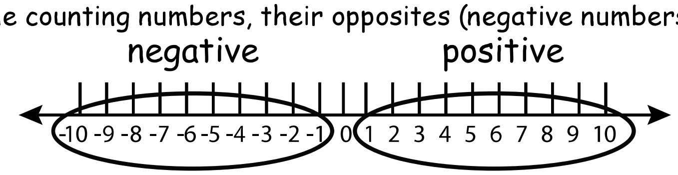


1. Find the area of the front: $l \times w$. (10 m x 5 m = 50 m^2)
   Since the back is identical, its area is the same.
2. Find the area of the top (C): $l \times h$. (10 m x 2 m = 20 m^2)
   Since the bottom (D) is identical, its area is the same.
3. Find the area of side A: $w \times h$. (2 m x 5 m = 10 m^2)
   Sinceside B is identical, its area is the same.
4. Add up the areas of all 6 faces.
   (10 m^2 + 10 m^2 + 20 m^2 + 20 m^2 + 50 m^2 + 50 m^2 = 160 m^2)

The formula is Surface Area = 2(length x width) + 2(length x height) + 2(width x height)
or $SA = 2lw + 2lh + 2wh$

Ordering Integers

Integers include the counting numbers, their opposites (negative numbers) and zero.



The negative numbers are to the left of zero. The positive numbers are to the right of zero.

The further a number is to the right, the greater its value. For example, 9 is further to the right than 2, so 9 is greater than 2.

In the same way, -1 is further to the right than -7, so -1 is greater than -7.

Examples: Order these integers from **least to greatest**: -10, 9, -25, 36, 0

Remember, the smallest number will be the one farthest to the left on the number line, -25, then -10, then 0. Next will be 9, and finally 36.

Answer: -25, -10, 0, 9, 36

Order these integers from **least to greatest**: -44, -19, -56, -80, -2

Answer: -80, -56, -44, -19, -2 (-80 is farthest to the left, so it is smallest.
-2 is farthest to the right, so it is greatest.)

Put these integers in order from **greatest to least**: -94, -6, -24, -70, -14
Now the greatest value (farthest to the right) will come first and the smallest value (farthest to the left) will come last.

Answer: -6, -14, -24, -70, -94

Help Pages

Solved Examples

Ratio and Proportion

A **ratio** is used to compare two numbers. There are three ways to write a ratio comparing 5 and 7.

1. Word form ➡ 5 to 7
2. Fraction form ➡ $\frac{5}{7}$
3. Ratio form ➡ 5 : 7

All are read as " five to seven."

You must make sure that all ratios are written in simplest form. (Just like fractions!!)

A **proportion** is a statement showing that two ratios are equal to each other. There are two ways to solve a proportion when a number is missing.

1. One way to solve a proportion is already familiar to you. You can use the equivalent fraction method.

$$\frac{5}{8} = \frac{n}{64} \quad (\times 8)$$

$n = 40$

So, $\frac{5}{8} = \frac{40}{64}$.

To use Cross-Products:

1. Multiply downward on each diagonal.
2. Make the product of each diagonal equal to each other.
3. Solve for the missing variable.

2. Another way to solve a proportion is by using cross-products.

$$\frac{14}{20} = \frac{21}{n}$$

$20 \times 21 = 14 \times n$

$420 = 14n$

$\frac{420}{14} = \frac{14n}{14}$

$30 = n$

So, $\frac{14}{20} = \frac{21}{30}$.

Percent

When changing from a fraction to a percent, a decimal to a percent, or from a percent to either a fraction or a decimal, it is very helpful to use an FDP chart (Fraction, Decimal, Percent).

To change a **fraction to a percent and/or decimal**, first find an equivalent fraction with 100 in the denominator. Once you have found that equivalent fraction, it can easily be written as a decimal. To change that decimal to a percent, move the decimal point 2 places to the right and add a % sign.

Example: Change $\frac{2}{5}$ to a percent and then to a decimal.

1. Find an equivalent fraction with 100 in the denominator.
2. From the equivalent fraction above, the decimal can easily be found. Say the name of the fraction: "forty hundredths." Write this as a decimal: 0.40.
3. To change 0.40 to a percent, move the decimal two places to the right. Add a % sign.

F	D	P
$\frac{2}{5}$		

F	D	P
$\frac{2}{5} = \frac{?}{100}$	0.40	

F	D	P
$\frac{2}{5} = \frac{?}{100}$	0.40	40%

$$\frac{2}{5} = \frac{?}{100} \quad (\times 20)$$

$? = 40$

$\frac{2}{5} = \frac{40}{100} = 0.40$

$0.40 = 40\%$

Help Pages

Solved Examples

Percent

When changing from a **percent to a decimal or a fraction**, the process is similar to the one used on the previous page. Begin with the percent. Write it as a fraction with a denominator of 100; reduce this fraction. Return to the percent, move the decimal point 2 places to the left. This is the decimal.

Example: Write 45% as a fraction and then as a decimal.

1. Begin with the percent. (45%) Write a fraction where the denominator is 100 and the numerator is the "percent." $\frac{45}{100}$
2. This fraction must be reduced. The reduced fraction is $\frac{9}{20}$.
3. Go back to the percent. Move the decimal point two places to the left to change it to a decimal.

F	D	P
$\frac{45(\div 5)}{100(\div 5)} = \frac{9}{20}$		45%

F	D	P
$\frac{9}{20}$		45% = 0.45

F	D	P
$\frac{9}{20}$	0.45	45%

When changing from a **decimal to a percent or a fraction**, again, the process is similar to the one used above. Begin with the decimal. Move the decimal point 2 places to the right and add a % sign. Return to the decimal. Write it as a fraction and reduce.

Example: Write 0.12 as a percent and then as a fraction.

1. Begin with the decimal. (0.12) Move the decimal point two places to the right to change it to a percent.
2. Go back to the decimal and write it as a fraction. Reduce this fraction.

F	D	P
	0.12	

F	D	P
	0.12 = 12%	12%

F	D	P
$\frac{12(\div 4)}{100(\div 4)} = \frac{3}{25}$	0.12	12%

Help Pages

Solved Examples

Compound Probability

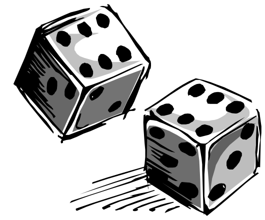
The **probability of two or more independent events** occurring together can be determined by multiplying the individual probabilities together. The product is called the compound probability.

Probability of A & B = (Probability of A) x (Probability of B)

or $P(A \text{ and } B) = P(A) \times P(B)$

Example: What is the probability of rolling a 6 and then a 2 on two rolls of a die [P(6 and 2)]?

A) First, find the probability of rolling a 6 [P(6)]. Since there are 6 numbers on a die and only one of them is a 6, the probability of getting a 6 is $\frac{1}{6}$.

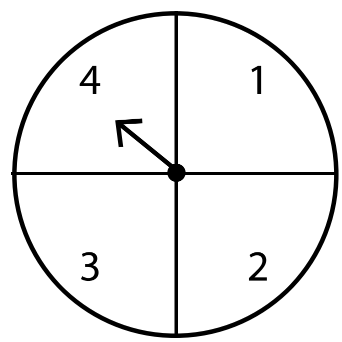


B) Then find the probability of rolling a 2 [P(2)]. Since there are 6 numbers on a die and only one of them is a 2, the probability of getting a 2 is $\frac{1}{6}$.

So, $P(6 \text{ and } 2) = P(6) \times P(2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

There is a 1 to 36 chance of getting a 6 and then a 2 on two rolls of a die.

Example: What is the probability of getting a 4 and then a number greater than 2 on two spins of this spinner [P(4 and greater than 2)]?



A) First, find the probability of getting a 4 [P(4)]. Since there are 4 numbers on the spinner and only one of them is a 4, the probability of getting a 4 is $\frac{1}{4}$.

B) Then find the probability of getting a number greater than 2 [P(greater than 2)]. Since there are 4 numbers on the spinner and two of them are greater than 2, the probability of getting a 2 is $\frac{2}{4}$.

So, $P(2 \text{ and greater than } 2) = P(2) \times P(\text{greater than } 2) = \frac{1}{4} \times \frac{2}{4} = \frac{2}{16} = \frac{1}{8}$

There is a 1 to 8 chance of getting a 4 and then a number greater than 2 on two spins of a spinner.

Example: On three flips of a coin, what is the probability of getting heads, tails, heads [P(H,T,H)]?

A) First, find the probability of getting heads [P(H)]. Since there are only 2 sides on a coin and only one of them is heads, the probability of getting heads is $\frac{1}{2}$.

B) Then find the probability of getting tails [P(T)]. Again, there are only 2 sides on a coin and only one of them is tails. The probability of getting tails is also $\frac{1}{2}$.

So, $P(H,T,H) = P(H) \times P(T) \times P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

There is a 1 to 8 chance of getting heads, tails and then heads on 3 flips of a coin.

Who Knows???

Degrees in a right angle? ....(90)

A straight angle? ....(180)

Angle greater than 90°? .... (obtuse)

Less than 90°? ....(acute)

Sides in a quadrilateral? ....(4)

Sides in an octagon?............................ (8)

Sides in a hexagon? ....(6)

Sides in a pentagon? .... (5)

Sides in a heptagon? ....(7)

Sides in a nonagon? .... (9)

Sides in a decagon? .... (10)

Inches in a yard? ....(36)

Yards in a mile? ....(1,760)

Feet in a mile? ....(5,280)

Centimeters in a meter? ....(100)

Teaspoons in a tablespoon? .... (3)

Ounces in a pound? ....(16)

Pounds in a ton?............................(2,000)

Cups in a pint? .... (2)

Pints in a quart? .... (2)

Quarts in a gallon? ....(4)

Millimeters in a meter? .... (1,000)

Years in a century? ....(100)

Years in a decade? ....(10)

Celsius freezing? ....(0°C)

Celsius boiling? ....(100°C)

Fahrenheit freezing? ....(32°F)

Fahrenheit boiling? ....(212°F)

Number with only 2 factors? .... (prime)

Perimeter? ....(add the sides)

Area? ....(length x width)

Volume? .... (length x width x height)

Area of parallelogram?............................ .... (base x height)

Area of triangle? ....($\frac{1}{2}$ base x height)

Area of trapezoid?.................................. ....($\frac{\text{base} + \text{base}}{2} \times$ height)

Surface area of a rectangular prism ....2(LW) + 2(WH) + 2(LH

Area of a circle? ....(πr^2)

Circumference of a circle? ....(πd)

Triangle with no sides equal? .... (scalene)

Triangle with 3 sides equal?................... ....(equilateral)

Triangle with 2 sides equal? .... (isosceles)

Distance across the middle of a circle? .... (diameter)

Half of the diameter? .... (radius)

Figures with the same size and shape? .... (congruent)

Figures with same shape, different sizes? ....(similar)

Number occurring most often? .... (mode)

Middle number? ....(median)

Answer in addition? ....(sum)

Answer in division? ....(quotient)

Answer in multiplication? .... (product)

Answer in subtraction? ......(difference)