

Minutes a Day—Mastery for a Lifetime!

Intermediate A

Mathematics

**Help Pages &
“Who Knows?”**









Help Pages

Vocabulary

Arithmetic Operations
difference — the result or answer to a subtraction problem. Example: The difference of 5 and 1 is 4.
product — the result or answer to a multiplication problem. Example: The product of 5 and 3 is 15.
quotient — the result or answer to a division problem. Example: The quotient of 8 and 2 is 4.
sum — the result or answer to an addition problem. Example: The sum of 5 and 2 is 7.
Factors and Multiples
factors — are multiplied together to get a product. Example: 2 and 3 are factors of 6.
multiples — can be evenly divided by a number. Example: 5, 10, 15 and 20 are multiples of 5.
composite number — a number with more than 2 factors. Example: 10 has factors of 1, 2, 5, and 10. Ten is a composite number.
prime number — a number with exactly 2 factors (the number itself and 1). Example: 7 has factors of 1 and 7. Seven is a prime number.
greatest common factor (GCF) — the highest factor that 2 numbers have in common. Example: The factors of 6 are 1, 2, 3, and 6. The factors of 9 are 1, 3, and 9. The GCF of 6 and 9 is 3.
least common multiple (LCM) — the smallest multiple that 2 numbers have in common. Example: Multiples of 3 are 3, 6, 9, 12, 15... Multiples of 4 are 4, 8, 12, 16... The LCM of 3 and 4 is 12.
prime factorization — a number, written as a product of its prime factors. Example: 140 can be written as $2 \times 2 \times 5 \times 7$ or $2^2 \times 5 \times 7$.
Fractions and Decimals
improper fraction — a fraction in which the numerator is larger than the denominator. Example: $\frac{9}{4}$
mixed number — the sum of a whole number and a fraction. Example: $5\frac{1}{4}$
reciprocal — a fraction where the numerator and denominator are interchanged. The product of a fraction and its reciprocal is always 1. Example: The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. $\frac{3}{5} \times \frac{5}{3} = \frac{15}{15} = 1$
repeating decimal — a decimal in which a number or a series of numbers continues on and on. Example: 2.33333333, 4.151515151515, 7.125555555, etc.
Geometry
acute angle — an angle measuring less than 90° .
congruent — figures with the same shape and the same size.
obtuse angle — an angle measuring more than 90° .
right angle — an angle measuring exactly 90° .
similar — figures having the same shape, but different size.
straight angle — an angle measuring exactly 180° .

Help Pages

Vocabulary (continued)

Geometry — Circles			
circumference — the distance around the outside of a circle.			
diameter — the widest distance across a circle. The diameter always passes through the center.			
radius — the distance from any point on the circle to the center. The radius is half of the diameter.			
Geometry — Polygons			
Number of Sides	Name	Number of Sides	Name
3 	triangle	7 	heptagon
4 	quadrilateral	8 	octagon
5 	pentagon	9 	nonagon
6 	hexagon	10 	decagon
Geometry — Triangles			
equilateral — a triangle in which all 3 sides have the same length.			
isosceles — a triangle in which 2 sides have the same length.			
scalene — a triangle in which no sides are the same length.			
Measurement — Relationships			
Volume		Distance	
3 teaspoons in a tablespoon		36 inches in a yard	
2 cups in a pint		1760 yards in a mile	
2 pints in a quart		5280 feet in a mile	
4 quarts in a gallon		100 centimeters in a meter	
Weight		Temperature	
16 ounces in a pound		0°Celsius - freezing point	
2000 pounds in a ton		100°Celsius - boiling point	
Time		32°Fahrenheit - freezing point	
10 years in a decade		212°Fahrenheit - boiling point	
100 years in a century			
Ratio and Proportion			
proportion — a statement that two ratios (or fractions) are equal. Example: $\frac{1}{2} = \frac{3}{6}$			
ratio — a comparison of two numbers by division; a ratio looks like a fraction. Example: $\frac{2}{5}$ or 2 to 5 or 2:5			

Help Pages

Vocabulary (continued)

Statistics

mean — the average of a group of numbers. The mean is found by finding the sum of a group of numbers and then dividing the sum by the number of members in the group.

Example: The average of 12, 18, 26, 17, and 22 is **19**.
$$\frac{12 + 18 + 26 + 17 + 22}{5} = \frac{95}{5} = 19$$

median — the middle value in a group of numbers. The median is found by listing the numbers in order from least to greatest, and finding the one that is in the middle of the list. If there is an even number of members in the group, the median is the average of the two middle numbers.

Example: The median of 14, 17, 24, 11, and 26 is **17**. 11, 14, (17), 24, 26

The median of 77, 93, 85, 95, 70, and 81 is **83**. 70, 77, (81, 85), 93, 95
$$\frac{81 + 85}{2} = 83$$

mode — the number that occurs most often in a group of numbers. The mode is found by counting how many times each number occurs in the list. The number that occurs more than any other is the mode. Some groups of numbers have more than one mode.

Example: The mode of 77, (93), 85, (93), 77, 81, (93), and 71 is **93**. (93 occurs more than the others.)

Place Value

Whole Numbers

8,	9	6	3,	2	7	1,	4	0	5
Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

The number above is read: eight billion, nine hundred sixty-three million, two hundred seventy-one thousand, four hundred five.

Decimal Numbers

1	7	8	.	6	4	0	5	9	2
Hundreds	Tens	Ones	↑ Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths

The number above is read: one hundred seventy-eight and six hundred forty thousand, five hundred ninety-two millionths.

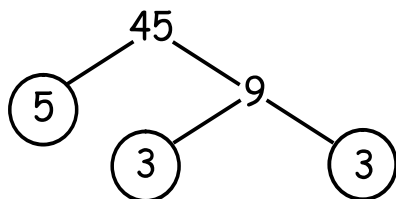
Help Pages

Solved Examples

Factors and Multiples

The **prime factorization** of a number is when a number is written as a product of its prime factors. A factor tree is helpful in finding the prime factors of a number.

Example: Use a factor tree to find the prime factors of 45.



1. Find any 2 factors of 45 (5 and 9).
2. If a factor is prime, circle it. If a factor is not prime, find 2 factors of it.
3. Continue until all factors are prime.
4. In the final answer, the prime factors are listed in order, least to greatest, using exponents when needed.

The prime factorization of 45 is $3 \times 3 \times 5$ or $3^2 \times 5$.

The **greatest common factor (GCF)** is the largest factor that two numbers have in common.

Example: Find the greatest common factor of 32 and 40.

The factors of 32 are 1, 2, 4, (8), 16, 32.

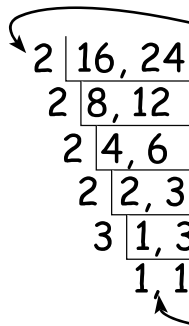
The factors of 40 are 1, 2, 4, 5, (8), 10, 20, 40.

The GCF of 32 and 40 is 8.

1. First list the factors of each number.
2. Find the largest number that is in both lists.

The **least common multiple (LCM)** is the smallest multiple that two numbers have in common. The prime factors of the numbers can be useful in finding the LCM.

Example: Find the least common multiple of 16 and 24.



1. If any of the numbers are even, factor out a 2.
2. Continue factoring out 2 until all numbers left are odd.
3. If the prime number cannot be divided evenly into the number, simply bring the number down.
4. Once you are left with all 1's at the bottom, you're finished!
5. Multiply all of the prime numbers (on the left side of the bracket) together to find the least common multiple.

The LCM of 16 and 24 is $2 \times 2 \times 2 \times 2 \times 3$ or 48.

Fractions

Changing from an **improper fraction** to a **mixed number**.

Example: Change the improper fraction $\frac{5}{2}$ to a mixed number.

$\frac{5}{2}$ (five halves) means $5 \div 2$.

So, $\frac{5}{2}$ is equal to 2 wholes and 1 half or $2\frac{1}{2}$.

$$\begin{array}{r} 2 \text{ wholes} \\ 2 \overline{)5} \\ \underline{-4} \\ 1 \text{ half} \end{array}$$

Help Pages

Solved Examples

Fractions (continued)

Changing from a **mixed number** to an **improper fraction**.

Example: Change the mixed number $7\frac{1}{4}$ to an improper fraction.

$$7\frac{1}{4} \quad 7 \times 4 = 28. \quad 28 + 1 = 29.$$

The new numerator is 29.

Keep the same denominator, 4.

The new fraction is $\frac{29}{4}$.

$7\frac{1}{4}$ is equal to $\frac{29}{4}$.

1. You're going to make a new fraction. To find the numerator of the new fraction, multiply the whole number by the denominator, and add the numerator
2. Keep the same denominator in your new fraction as you had in the mixed number.

Equivalent fractions are two fractions that are equal to each other. Usually you will be finding a missing numerator or denominator.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a denominator of 35.

$$\begin{array}{c} \times 7 \\ \curvearrowright \\ \frac{4}{5} = \frac{?}{35} \\ \curvearrowleft \\ \times 7 \end{array}$$

1. Ask yourself, "What did I do to 5 to get 35?" (Multiply by 7.)
2. Whatever you did in the denominator, you also must do in the numerator. $4 \times 7 = 28$. The missing numerator is 28.

So, $\frac{4}{5}$ is equivalent to $\frac{28}{35}$.

Example: Find a fraction that is equivalent to $\frac{4}{5}$ and has a numerator of 24.

$$\begin{array}{c} \times 6 \\ \curvearrowright \\ \frac{4}{5} = \frac{24}{?} \\ \curvearrowleft \\ \times 6 \end{array}$$

1. Ask yourself, "What did I do to 4 to get 24?" (Multiply by 6.)
2. Whatever you did in the numerator, you also must do in the denominator. $5 \times 6 = 30$. The missing denominator is 30.

So, $\frac{4}{5}$ is equivalent to $\frac{24}{30}$.

Comparing fractions means looking at two or more fractions and determining if they are equal, if one is greater than ($>$) the other, or if one is less than ($<$) the other. A simple way to compare fractions is by cross-multiplying, using the steps below.

Examples: Compare these fractions. Use the correct symbol. $\frac{8}{9} \bigcirc \frac{3}{4}$ $\frac{7}{9} \bigcirc \frac{6}{7}$

$$\begin{array}{c} 32 > 27 \\ \swarrow \quad \searrow \\ \frac{8}{9} \bigcirc \frac{3}{4} \\ \swarrow \quad \searrow \\ 24 < 27 \end{array}$$

$$\begin{array}{c} 49 < 54 \\ \swarrow \quad \searrow \\ \frac{7}{9} \bigcirc \frac{6}{7} \\ \swarrow \quad \searrow \\ 42 < 48 \end{array}$$

So, $\frac{8}{9} > \frac{3}{4}$ and $\frac{7}{9} < \frac{6}{7}$

1. Begin with the denominator on the left and multiply by the opposite numerator. Put the answer (product) above the side where you ended. ($9 \times 3 = 27$)
2. Cross-multiply the other denominator and numerator and put that product above where you ended.
3. Compare the two products and insert the correct symbol.

HINT: Always multiply diagonally upwards!

Help Pages

Solved Examples

Fractions (continued)

To add (or subtract) fractions with the same denominator, simply add (or subtract) the numerators, keeping the same denominator.

Examples: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$$\frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

To add mixed numbers, follow a process similar to the one you used with fractions. If the sum is an improper fraction, be sure to simplify it.

Example: $1\frac{2}{5} + 1\frac{4}{5}$

$$\begin{array}{r} 1\frac{2}{5} \\ +1\frac{4}{5} \\ \hline 2\frac{6}{5} \end{array}$$

$2\frac{6}{5}$ is improper. $\frac{6}{5}$ can be rewritten as $1\frac{1}{5}$.

So, $2\frac{6}{5}$ is $2 + 1\frac{1}{5} = 3\frac{1}{5}$.

When adding fractions that have different denominators, you need to change the fractions so they have a common denominator before they can be added.

Finding the least common denominator (LCD):

The LCD of the fractions is the same as the least common multiple of the denominators. Sometimes, the LCD will be the product of the denominators.

Example: Find the sum of $\frac{3}{8}$ and $\frac{1}{12}$.

$$\begin{array}{r} \frac{3}{8} = \frac{9}{24} \\ + \frac{1}{12} = \frac{2}{24} \\ \hline \frac{11}{24} \end{array}$$

1. First, find the LCM of 8 and 12.
2. The LCM of 8 and 12 is 24. This is also the LCD of these 2 fractions.
3. Find an equivalent fraction for each that has a denominator of 24.
4. When they have a common denominator, the fractions can be added.

$$\begin{array}{r} 2 \overline{) 8, 12} \\ 2 \overline{) 4, 6} \\ 2 \overline{) 2, 3} \\ 3 \overline{) 1, 3} \\ 1, 1 \end{array} \quad 2 \times 2 \times 2 \times 3 = 24$$

The LCM is 24.
So, the LCD is 24.

Example: Add $\frac{1}{4}$ and $\frac{1}{5}$.

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ + \frac{1}{5} = \frac{4}{20} \\ \hline \frac{9}{20} \end{array}$$

$$4 \times 5 = 20 \quad \text{The LCM is 20.}$$

When adding mixed numbers with unlike denominators, follow a process similar to the one you used with fractions (above). Be sure to put your answer in simplest form.

Example: Find the sum of $6\frac{3}{7}$ and $5\frac{2}{3}$.

$$\begin{array}{r} 6\frac{3}{7} = 6\frac{9}{21} \\ + 5\frac{2}{3} = 5\frac{14}{21} \\ \hline 11\frac{23}{21} \end{array} \quad \frac{23}{21} = 1\frac{2}{21} + 11 = 12\frac{2}{21}$$

(improper)

1. Find the LCD.
2. Find the missing numerators.
3. Add the whole numbers, then add the fractions.
4. Make sure your answer is in simplest form.

Help Pages

Solved Examples

Fractions (continued)

When subtracting numbers with unlike denominators, follow a process similar to the one you used when adding fractions. Be sure to put your answer in simplest form.

Examples: Find the difference of $\frac{3}{4}$ and $\frac{2}{5}$.

Subtract $\frac{1}{16}$ from $\frac{3}{8}$.

$$\begin{array}{r} \frac{3}{4} = \frac{15}{20} \\ - \frac{2}{5} = \frac{8}{20} \\ \hline \frac{7}{20} \end{array}$$

1. Find the LCD just as you did when adding fractions.
2. Find the missing numerators.
3. Subtract the numerators and keep the common denominator.
4. Make sure your answer is in simplest form.

$$\begin{array}{r} \frac{3}{8} = \frac{6}{16} \\ - \frac{1}{16} = \frac{1}{16} \\ \hline \frac{5}{16} \end{array}$$

When subtracting mixed numbers with unlike denominators, follow a process similar to the one you used when adding mixed numbers. Be sure to put your answer in simplest form.

Example: Subtract $4\frac{2}{5}$ from $8\frac{9}{10}$.

1. Find the LCD.
2. Find the missing numerators.
3. Subtract and simplify your answer.

$$\begin{array}{r} 8\frac{9}{10} = 8\frac{9}{10} \\ - 4\frac{2}{5} = 4\frac{4}{10} \\ \hline 4\frac{5}{10} = 4\frac{1}{2} \end{array}$$

Sometimes when subtracting mixed numbers, you may need to rename them. If the numerator of the top fraction is smaller than the numerator of the bottom fraction, rename the mixed number as in the example below.

Example: Subtract $5\frac{5}{6}$ from $9\frac{1}{4}$.

1. Find the LCD.
2. Find the missing numerators. Write equivalent fractions.
3. Because you can't subtract 10 from 3, rename the whole number as a mixed number using the common denominator.
4. Add the two fractions to get an improper fraction.
5. Subtract the whole numbers and the fractions and simplify your answer.

$$\begin{array}{r} 9\frac{1}{4} = 9\frac{3}{12} = 8\frac{12}{12} + \frac{3}{12} = 8\frac{15}{12} \\ - 5\frac{5}{6} = 5\frac{10}{12} \\ \hline 3\frac{5}{12} \end{array}$$

More examples:

$$\begin{array}{r} 8\frac{1}{2} = 8\frac{2}{4} = 7\frac{4}{4} + \frac{2}{4} = 7\frac{6}{4} \\ - 4\frac{3}{4} = 4\frac{3}{4} \\ \hline 3\frac{3}{4} \end{array}$$

$$\begin{array}{r} 10\frac{1}{5} = 10\frac{4}{20} = 9\frac{20}{20} + \frac{4}{20} = 9\frac{24}{20} \\ - 6\frac{3}{4} = 6\frac{15}{20} \\ \hline 3\frac{9}{20} \end{array}$$

Help Pages

Solved Examples

Fractions (continued)

To **multiply fractions**, simply multiply the numerators together to get the numerator of the product. Then multiply the denominators together to get the denominator of the product. Make sure your answer is in simplest form.

Examples: Multiply $\frac{3}{5}$ by $\frac{2}{3}$.

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$$

1. Multiply the numerators.
2. Multiply the denominators.
3. Simplify your answer.

Multiply $\frac{5}{8}$ by $\frac{4}{5}$.

$$\frac{5}{8} \times \frac{4}{5} = \frac{20}{40} = \frac{1}{2}$$

Sometimes you can use cancelling when multiplying fractions. Let's look at the examples again.

$$\overset{1}{\cancel{3}} \frac{2}{5} \times \frac{2}{\cancel{3}_1} = \frac{2}{5}$$

The 3s have a common factor — 3. Divide both of them by 3. Since, $3 \div 3 = 1$, we cross out the 3s and write 1s in their place.

Now, multiply the fractions. In the numerator, $1 \times 2 = 2$. In the denominator, $5 \times 1 = 5$.

The answer is $\frac{2}{5}$.

1. Are there any numbers in the numerator and the denominator that have common factors?
2. If so, cross out the numbers, divide both by that factor, and write the quotient.
3. Then, multiply the fractions as described above, using the quotients instead of the original numbers.

$$\overset{1}{\cancel{5}} \frac{4}{8} \times \frac{\cancel{4}^1}{\cancel{5}_1} = \frac{1}{2}$$

As in the other example, the 5s can be cancelled.

But here, the 4 and the 8 also have a common factor — 4.

$$8 \div 4 = 2 \text{ and } 4 \div 4 = 1.$$

After cancelling both of these, you can multiply the fractions.

REMEMBER: You can cancel up and down or diagonally, but NEVER sideways!

When **multiplying mixed numbers**, you must first change them into improper fractions.

Examples: Multiply $2\frac{1}{4}$ by $3\frac{1}{9}$.

$$2\frac{1}{4} \times 3\frac{1}{9} = \overset{1}{\cancel{4}} \frac{9}{1} \times \frac{\cancel{28}^7}{\cancel{9}_1} = \frac{7}{1} = 7$$

1. Change each mixed number to an improper fraction.
2. Cancel wherever you can.
3. Multiply the fractions.
4. Put your answer in simplest form.

Multiply $3\frac{1}{8}$ by 4.

$$3\frac{1}{8} \times 4 = \frac{\overset{1}{\cancel{25}}}{\cancel{8}_2} \times \frac{\cancel{4}^1}{1} = \frac{25}{2} = 12\frac{1}{2}$$

To **divide fractions**, you must take the reciprocal of the 2nd fraction, and then multiply that reciprocal by the 1st fraction. Don't forget to simplify your answer!

Examples: Divide $\frac{1}{2}$ by $\frac{7}{12}$.

$$\frac{1}{2} \div \frac{7}{12} = \frac{1}{\cancel{2}_1} \times \frac{\cancel{12}^6}{7} = \frac{6}{7}$$

1. Keep the 1st fraction as it is.
2. Write the reciprocal of the 2nd fraction.
3. Change the sign to multiplication.
4. Cancel if you can and multiply.
5. Simplify your answer.

Divide $\frac{7}{8}$ by $\frac{3}{4}$.

$$\frac{7}{8} \div \frac{3}{4} = \frac{7}{\cancel{8}_2} \times \frac{\cancel{4}^1}{3} = \frac{7}{6} = 1\frac{1}{6}$$

Help Pages

Solved Examples

Fractions (continued)

When dividing mixed numbers, you must first change them into improper fractions.

Example: Divide $1\frac{1}{4}$ by $3\frac{1}{2}$.

$$\begin{aligned} 1\frac{1}{4} \div 3\frac{1}{2} &= \\ \frac{5}{4} \div \frac{7}{2} &= \\ \frac{5}{\cancel{4}^2} \times \frac{\cancel{2}^1}{7} &= \frac{5}{14} \end{aligned}$$

1. Change each mixed number to an improper fraction.
2. Keep the 1st fraction as it is.
3. Write the reciprocal of the 2nd fraction.
4. Change the sign to multiplication.
5. Cancel if you can and multiply.
6. Simplify your answer.

Decimals

When we compare decimals, we are looking at two or more decimal numbers and deciding which has the smaller or larger value. We sometimes compare by placing them in order from least to greatest or from greatest to least. Another way to compare is to use the symbols for "less than" (<), "greater than" (>), or "equal to" (=).

Example: Order these numbers from least to greatest. 0.561 0.506 0.165

1. Write the numbers in a column, lining up the decimal points.
2. Write zeroes, if necessary, so all have the same number of digits.
3. Begin on the left and compare the digits.

0.561

0.506

0.165

Since they all have 3 digits, we don't need to add zeroes.

Beginning on the left, the fives are equal, but the one is less, so 0.165 is the smallest.

Then, look at the next digit. The zero is less than the six, so 0.506 is next smallest.

So, in order from least to greatest:

0.165, 0.506, 0.561

Example: Place these numbers in order from greatest to least. 0.44 0.463 0.045

0.440

0.463

0.045

After lining up the numbers, we must add a zero to 0.44 to make them all have the same number of digits.

Beginning on the left, the zero is smaller than the fours, so 0.045 is the smallest.

Look at the next digit. The four is smaller than the six, so 0.440 is the next smallest.

In order from greatest to least: 0.463, 0.440, 0.045

Help Pages

Solved Examples

Decimals (continued)

When we **round decimals**, we are approximating them. This means we end the decimal at a certain place value and we decide if it's closer to the next higher number (round up) or to the next lower number (keep the same). It might be helpful to look at the decimal place-value chart on p. 286.

Example: Round 0.574 to the tenths place.

There is a 5 in the rounding (tenths) place.

Since 7 is greater than 5, change the 5 to a 6.

Drop the digits to the right of the tenths place.

0.574

0.574

0.6

1. Identify the number in the rounding place.
2. Look at the digit to its right.
3. If the digit is 5 or greater, increase the number in the rounding place by 1. If the digit is less than 5, keep the number in the rounding place the same.
4. Drop all digits to the right of the rounding place.

Example: Round 2.783 to the nearest hundredth.

2.783

2.783

2.78

There is an 8 in the rounding place.

Since 3 is less than 5, keep the rounding place the same.

Drop the digits to the right of the hundredths place.

Adding and subtracting decimals is very similar to adding or subtracting whole numbers. The main difference is that you have to line-up the decimal points in the numbers before you begin.

Examples: Find the sum of 3.14 and 1.2.

Add 55.1, 6.472, and 18.33.

$$\begin{array}{r} 3.14 \\ + 1.20 \\ \hline 4.34 \end{array}$$

1. Line up the decimal points. Add zeroes as needed.
2. Add (or subtract) the decimals.
3. Add (or subtract) the whole numbers.
4. Bring the decimal point straight down.

$$\begin{array}{r} 55.100 \\ 6.472 \\ +18.330 \\ \hline 79.902 \end{array}$$

Examples: Subtract 3.7 from 9.3.

Find the difference of 4.1 and 2.88.

$$\begin{array}{r} 9.3 \\ - 3.7 \\ \hline 5.6 \end{array}$$

$$\begin{array}{r} 4.10 \\ -2.88 \\ \hline 1.22 \end{array}$$

Help Pages

Solved Examples

Decimals (continued)

When multiplying a decimal by a whole number, the process is similar to multiplying whole numbers.

Examples: Multiply 3.42 by 4.

$$\begin{array}{r} 3.42 \longrightarrow 2 \text{ decimal places} \\ \times 4 \longrightarrow 0 \text{ decimal places} \\ \hline 13.68 \longrightarrow \text{Place decimal point so} \\ \text{there are 2 decimal} \\ \text{places.} \end{array}$$

1. Line up the numbers on the right.
2. Multiply. Ignore the decimal point.
3. Place the decimal point in the product. (The total number of decimal places in the product must equal the total number of decimal places in the factors.)

Find the product of 2.3 and 2.

$$\begin{array}{r} 2.3 \longrightarrow 1 \text{ decimal place} \\ \times 2 \longrightarrow 0 \text{ decimal places} \\ \hline 4.6 \longrightarrow \text{Place decimal point so} \\ \text{there is 1 decimal place.} \end{array}$$

The process for multiplying two decimal numbers is a lot like what we just did above.

Examples: Multiply 0.4 by 0.6.

$$\begin{array}{r} 0.4 \longrightarrow 1 \text{ decimal place} \\ \times 0.6 \longrightarrow 1 \text{ decimal place} \\ \hline 0.24 \longrightarrow \text{Place decimal point so} \\ \text{there are 2 decimal places.} \end{array}$$

Find the product of 2.67 and 0.3.

$$\begin{array}{r} 2.67 \longrightarrow 2 \text{ decimal places} \\ \times 0.3 \longrightarrow 1 \text{ decimal place} \\ \hline 0.801 \longrightarrow \text{Place decimal point so} \\ \text{there are 3 decimal places.} \end{array}$$

Sometimes it is necessary to add zeroes in the product as placeholders in order to have the correct number of decimal places.

Example: Multiply 0.03 by 0.4.

$$\begin{array}{r} 0.03 \longrightarrow 2 \text{ decimal places} \\ \times 0.4 \longrightarrow 1 \text{ decimal place} \\ \hline 0.012 \longrightarrow \text{Place decimal point so} \\ \text{there are 3 decimal places.} \end{array}$$

We had to add a zero in front of the 12 so that we could have 3 decimal places in the product.

The process for dividing a decimal number by a whole number is similar to dividing whole numbers.

Examples: Divide 6.4 by 8.

$$\begin{array}{r} 0.8 \\ 8 \overline{)6.4} \\ \underline{-64} \\ 0 \end{array}$$

1. Set up the problem for long division.
2. Place the decimal point in the quotient directly above the decimal point in the dividend.
3. Divide. Add zeroes as placeholders if necessary. (See examples below.)

Find the quotient of 20.7 and 3.

$$\begin{array}{r} 6.9 \\ 3 \overline{)20.7} \\ \underline{-18} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

Examples: Divide 4.5 by 6.

$$\begin{array}{r} 0.75 \\ 6 \overline{)4.50} \\ \underline{-42} \downarrow \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

Add zero(es).

Bring zero down.

Keep dividing.

Find the quotient of 3.5 and 4.

$$\begin{array}{r} 0.875 \\ 4 \overline{)3.500} \\ \underline{-32} \downarrow \downarrow \\ 30 \downarrow \\ \underline{-28} \downarrow \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Help Pages

Solved Examples

Decimals (continued)

When dividing decimals the remainder is not always zero. Sometimes, the division continues on and on and the remainder begins to repeat itself. When this happens the quotient is called a **repeating decimal**.

Examples: Divide 2 by 3.

$$\begin{array}{r} 0.\overline{66} \\ 3 \overline{) 2.000} \\ \underline{- 18} \downarrow \downarrow \\ 20 \downarrow \\ \underline{- 18} \downarrow \\ 20 \end{array}$$

Divide 10 by 11.

$$\begin{array}{r} 0.90\overline{90} \\ 11 \overline{) 10.00000} \\ \underline{- 99} \downarrow \downarrow \downarrow \\ 100 \downarrow \downarrow \\ \underline{- 99} \downarrow \downarrow \\ 100 \end{array}$$

← Add zeroes as needed →

← This pattern (with the same remainder begins to repeat itself. →

To write the final answer, put a bar in the quotient over the digits that repeat.

The process for **dividing a decimal number by a decimal number** is similar to other long division that you have done. The main difference is that we have to move the decimal point in both the dividend and the divisor the same number of places to the right.

Examples: Divide 1.8 by 0.3.

$$\begin{array}{r} 6 \\ 0.3 \overline{) 1.8} \\ \underline{- 18} \\ 0 \end{array}$$

1. Change the divisor to a whole number by moving the decimal point as many places to the right as needed.
2. Move the decimal in the dividend the same number of places to the right as you did in the divisor.
3. Put the decimal point in the quotient directly above the decimal point in the dividend.
4. Divide.

Divide 0.385 by 0.05.

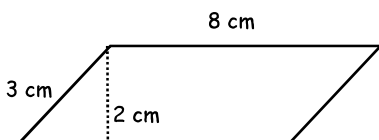
$$\begin{array}{r} 7.7 \\ 0.05 \overline{) 0.385} \\ \underline{- 35} \downarrow \\ 35 \\ \underline{- 35} \\ 0 \end{array}$$

Geometry

Finding the **area of a parallelogram** is similar to finding the area of any other quadrilateral. The area of the figure is equal to the length of its base multiplied by the height of the figure.

Area of parallelogram = base × height or $A = b \times h$

Example: Find the area of the parallelogram below.



1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down - never slanted.)
3. Multiply to find the area. (16 cm²)

So, $A = 8 \text{ cm} \times 2 \text{ cm} = 16 \text{ cm}^2$.

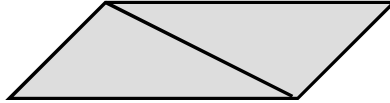
Help Pages

Solved Examples

Geometry (continued)

To find the **area of a triangle**, it is helpful to recognize that any triangle is exactly half of a parallelogram.

The whole figure is a parallelogram.

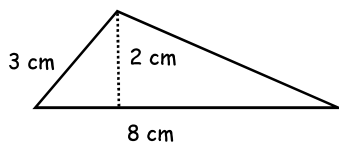


Half of the whole figure is a triangle.

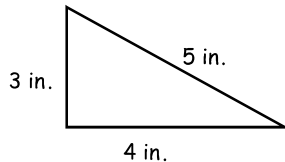
So, the triangle's area is equal to half of the product of the base and the height.

$$\text{Area of triangle} = \frac{1}{2}(\text{base} \times \text{height}) \quad \text{or} \quad A = \frac{1}{2}bh \quad \text{or} \quad A = \frac{bh}{2}$$

Examples: Find the area of the triangles below.



$$\text{So, } A = 8 \text{ cm} \times 2 \text{ cm} \times \frac{1}{2} = 8 \text{ cm}^2.$$



$$\text{So, } A = 4 \text{ in.} \times 3 \text{ in.} \times \frac{1}{2} = 6 \text{ in}^2.$$

1. Find the length of the base. (8 cm)
2. Find the height. (It is 2 cm. The height is always straight up and down - never slanted.)
3. Multiply them together and divide by 2 to find the area. (8 cm²)

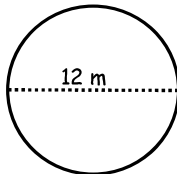
The base of this triangle is 4 inches long. Its height is 3 inches. (Remember the height is always straight up and down!)

The **circumference of a circle** is the distance around the outside of the circle. Before you can find the circumference of a circle you must know either its radius or its diameter. Also, you must know the value of the constant, pi (π). $\pi = 3.14$ (rounded to the nearest hundredth)

Once you have this information, the circumference can be found by multiplying the diameter by pi.

$$\text{Circumference} = \pi \times \text{diameter} \quad \text{or} \quad C = \pi d$$

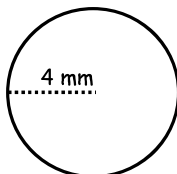
Examples: Find the circumference of the circles below.



$$\text{So, } C = 12 \text{ m} \times 3.14 = 37.68 \text{ m.}$$

1. Find the length of the diameter. (12 m)
2. Multiply the diameter by π . (12 m \times 3.14)
3. The product is the circumference. (37.68 m)

Sometimes the radius of a circle is given instead of the diameter. Remember, the radius of any circle is exactly half of the diameter. If a circle has a radius of 3 feet, its diameter is 6 feet.



$$\text{So, } C = 8 \text{ mm} \times 3.14 = 25.12 \text{ mm.}$$

1. Since the radius is 4 mm, the diameter must be 8 mm.
2. Multiply the diameter by π . (8 mm \times 3.14)
3. The product is the circumference. (25.12 mm)

Help Pages

Solved Examples

Ratio and Proportion

A **ratio** is used to compare two numbers. There are three ways to write a ratio comparing 5 and 7.

1. Word form → 5 to 7
2. Fraction form → $\frac{5}{7}$
3. Ratio form → 5 : 7

You must make sure that all ratios are written in simplest form. (Just like fractions!!)

A **proportion** is a statement showing that two ratios are equal to each other. There are two ways to solve a proportion when a number is missing.

1. One way to solve a proportion is already familiar to you. You can use the equivalent fraction method.

$$\begin{array}{c} \times 8 \\ \curvearrowright \\ \frac{5}{8} = \frac{n}{64} \\ \curvearrowleft \\ \times 8 \\ n = 40 \\ \text{So, } \frac{5}{8} = \frac{40}{64} \end{array}$$

To use Cross-Products:

1. Multiply downward on each diagonal.
2. Make the product of each diagonal equal to each other.
3. Solve for the missing variable.

2. Another way to solve a proportion is by using cross-products.

$$\begin{array}{c} \frac{14}{20} = \frac{21}{n} \\ \swarrow \quad \searrow \\ 20 \times 21 = 14 \times n \\ 420 = 14n \\ \frac{420}{14} = \frac{14n}{14} \\ 30 = n \\ \text{So, } \frac{14}{20} = \frac{21}{30} \end{array}$$

Who Knows???

Degrees in a right angle?(90)	Perimeter?(add the sides)
A straight angle?(180)	Area?(length x width)
Angle greater than 90°? (obtuse)	Volume? (length x width x height)
Less than 90°?(acute)	Area of parallelogram?..... (base x height)
Sides in a quadrilateral?(4)	Area of triangle?($\frac{1}{2}$ base x height)
Sides in an octagon?..... (8)	Area of trapezoid?($\frac{\text{base} + \text{base}}{2} \times \text{height}$)
Sides in a hexagon?(6)	Area of a circle?(πr^2)
Sides in a pentagon? (5)	Circumference of a circle?(πd)
Sides in a heptagon? (7)	Triangle with no sides equal? (scalene)
Sides in a nonagon? (9)	Triangle with 3 sides equal?..... (equilateral)
Sides in a decagon? (10)	Triangle with 2 sides equal? (isosceles)
Inches in a yard? (36)	Distance across the middle of a circle? (diameter)
Yards in a mile?(1,760)	Half of the diameter? (radius)
Feet in a mile? (5,280)	Figures with the same size and shape? (congruent)
Centimeters in a meter? (100)	Figures with same shape, different sizes?(similar)
Teaspoons in a tablespoon? (3)	Number occurring most often? (mode)
Ounces in a pound?(16)	Middle number?(median)
Pounds in a ton?(2,000)	Answer in addition?(sum)
Cups in a pint? (2)	Answer in division?(quotient)
Pints in a quart? (2)	Answer in subtraction? (difference)
Quarts in a gallon?(4)	Answer in multiplication? (product)
Millimeters in a meter? (1,000)	
Years in a century?(100)	
Years in a decade?(10)	
Celsius freezing? (0°C)	
Celsius boiling?(100°C)	
Fahrenheit freezing?(32°F)	
Fahrenheit boiling?(212°F)	
Number with only 2 factors? (prime)	