

# Level 4

3<sup>rd</sup> Edition

# Mathematics

**Help Pages** 

## Vocabulary

Arithmetic Operations						
difference — the result or answer to a subtraction problem. Example: The difference of 5 and 1 is 4.						
product — the result or answer to a multiplication problem. Example: The product of 5 and 3 is 15.						
quotient — the result or answer to a division problem. Example: The quotient of 8 and 2 is 4.						
sum — the result or answ	er to an addition problem.	<b>Example</b> : The sum of 5 and	2 is 7.			
Fractions						
<b>denominator</b> — the bottom number of a fraction. <b>Example</b> : $\frac{1}{4}$ ; the denominator is 4						
<b>fraction</b> — a part of a whole. <b>Example</b> : This box has 4 parts. 1 part is shaded. $\frac{1}{4}$ is shaded.						
improper fraction — a fraction in which the numerator is larger than the denominator. Example: $\frac{9}{4}$						
<b>mixed number</b> — the sum of a whole number and a fraction. <b>Example</b> : $5 \frac{1}{4}$						
<b>numerator</b> — the top number of a fraction. <b>Example</b> : $\frac{1}{4'}$ The numerator is 1.						
Geometry						
acute angle — an angle measuring less than 90°.						
area — the size of a surface. Area is always given in square units (feet², meters²,).						
circumference — the distance around the outside of a circle.						
congruent — figures with the same shape and the same size.						
diameter — the widest distance across a circle. The diameter always passes through the center.						
line of symmetry — a line along which a figure can be folded so that the two halves match exactly.						
obtuse angle — an angle measuring more than 90°.						
perimeter — the distance around the outside of a polygon.						
radius — the distance from any point on the circle to the center. The radius is half of the diameter.						
right angle — an angle measuring exactly 90°.						
similar — figures having the same shape, but different sizes.						
straight angle — an angle measuring exactly 180°.						
<b>volume</b> — the measure of space inside a solid figure. Volume is expressed in cubic units $(m^3, ft.^3,)$ .						
Geometry — Polygons						
Number of Sides	Name	Number of Sides	Name			
з 🛆	triangle	6	hexagon			
4 🗆	quadrilateral	8	octagon			
5	pentagon					

#### Vocabulary

Measurement — Relationships					
Volume	Distance				
3 teaspoons in a tablespoon	36 inches in a yard				
2 cups in a pint	1760 yards in a mile				
2 pints in a quart	5280 feet in a mile				
4 quarts in a gallon	100 centimeters in a meter				
Weight	1000 millimeters in a meter				
16 ounces in a pound	Temperature				
2000 pounds in a ton	0° Celsius - Freezing Point				
Time	100° Celsius - Boiling Point				
10 years in a decade	32° Fahrenheit - Freezing Point				
100 years in a century	212° Fahrenheit – Boiling Point				

#### **Statistics**

**Mean** — the average of a group of numbers. The mean is found by finding the sum of a group of numbers and then dividing the sum by the number of members in the group.

**Example:** The average of 12, 18, 26, 17 and 22 is 19.

$$\frac{12+18+26+17+22}{5} = \frac{95}{5} = 19$$

Median — the middle value in a group of numbers. The median is found by listing the numbers in order from least to greatest and finding the one that is in the middle of the list. If there is an even number of members in the group, the median is the average of the two middle numbers.

Example: The median of 14, 17, 24, 11 and 26 is 17. 11, 14, 17, 24, 26

The median of 77, 93, 85, 95, 70 and 81 is 83. 70, 77, 81, 85, 93, 95, 81 = 83

$$\frac{81+85}{2}=83$$

Mode — the number that occurs most often in a group of numbers. The mode is found by counting how many times each number occurs in the list. The number that occurs more than any other is the mode. Some groups of numbers have more than one mode.

**Example**: The mode of 77, (93), 85, (93), 77, 81, (93), and 71 is 93. (93 occurs more than the others.)

Range — the difference between the largest number and the smallest number in a set of data. To find the range, subtract the smallest number from the largest number.

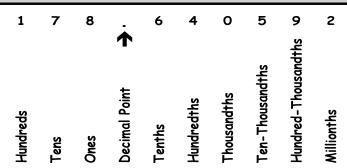
**Example:** The range of 77, 93, 85, 95, 70 and 81 is 25. (95 - 70 = 25)

#### Place Value

Whole Numbers												
	8,	9	6	3,	2	7	1,	4	0	5		
	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones		

The number above is read: eight billion, nine hundred sixty-three million, two hundred seventy-one thousand, four hundred five.

#### Decimals



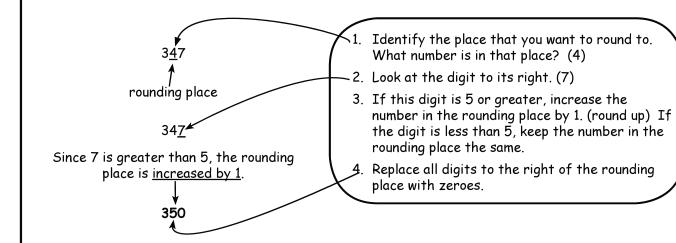
The number above is read: one hundred seventy-eight and six hundred forty thousand, five hundred ninety-two millionths.

#### Solved Examples

#### Whole Numbers

When we **round numbers**, we are estimating them. This means we focus on a particular place value, and decide if that digit is closer to the next highest number (round up) or to the next lower number (keep the same). It might be helpful to look at the place-value chart above.

Example: Round 347 to the tens place.

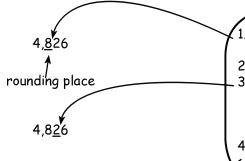


## Solved Examples

#### Whole Numbers (continued)

Here is another example of rounding whole numbers.

Example: Round 4,826 to the hundreds place.



Since 2 is less than 5, the rounding place stays the same.

4.800

Identify the place that you want to round to. What number is in that place? (8)

2. Look at the digit to its right.

If this digit is 5 or greater, increase the number in the rounding place by 1. (round up) If the digit is less than 5, keep the number in the rounding place the same.

Replace all digits to the right of the rounding place with zeroes.

Round 27,934 to the thousands place.

 $27,934 \rightarrow 7$  is in the rounding place.

 $27,934 \longrightarrow 9$  is greater than 5, so the rounding place will go up by 1.

**28,000**  $\longrightarrow$  The digits to the right of the rounding place are changed to zeroes.

When adding or subtracting whole numbers, first the numbers must be lined-up on the right. Starting with the ones place, add (or subtract) the numbers; when adding, if the answer has 2 digits, write the ones digit and regroup the tens digit (for subtraction, it may also be necessary to regroup first). Then, add (or subtract) the numbers in the tens place. Continue with the hundreds, etc.

Look at these examples of addition.

**Examples:** Find the sum of 314 and 12.

Add 6,478 and 1,843.

 $\frac{314}{+12}$ 

1. Line up the numbers on the right.

Beginning with the ones place, add. Regroup if necessary.

3. Repeat with the tens place.

4. Continue this process with the hundreds place, etc.

6,478+1,843 $\hline 8,321$ 

## Solved Examples

#### Whole Numbers (continued)

Use the following examples of subtraction to help you.

Examples: Subtract 37 from 93.

- 1. Begin with the ones place. Check to see if you need to regroup. Since 7 is larger than 3, you must regroup to 8 tens and 13 ones.
- 2. Now look at the tens place. Since 3 is less than 8, you do not need to regroup.
- 3. Subtract each place value beginning with the ones.

Find the difference of 4,125 and 2,033.

$$4,1/2/5$$

$$-2,033$$

$$2,092$$

- 1. Begin with the ones place. Check to see if you need to regroup. Since 3 is less than 5, you do not need to regroup.
- 2. Now look at the tens place. Check to see if you need to regroup. Since 3 is larger than 2, you must regroup to 3 hundreds and 12 tens.
- 3. Now look at the hundreds place. Since 2 is less than 4, you are ready to subtract.
- 4. Subtract each place value beginning with the ones.

Sometimes when doing subtraction, you must subtract from zero. This always requires regrouping. Use the examples below to help you.

Examples: Subtract 2,361 from 5,000.

- 1. Begin with the ones place. Since 1 is larger than 0, you must regroup. You must continue to the thousands place, and then begin regrouping.
- 2. Regroup the thousands place to 4 thousands and 10 hundreds.
- 3. Next regroup the hundreds place to 9 hundreds and 10 tens.
- 4. Then, regroup the tens place to 9 tens and 10 ones.
- 5. Finally, subtract each place value beginning with the ones.

Find the difference between 600 and 238.

#### Solved Examples

#### Whole Numbers (continued)

When multiplying multi-digit whole numbers, it is important to know your multiplication facts. Follow the steps and the examples below.

Examples: Multiply 23 by 5.

- 1. Line up the numbers on the right.
- 2. Multiply the digits in the ones place. Regroup if necessary.
- 3. Multiply the digits in the tens place. Add any regrouped tens.
- 4. Repeat step 3 for the hundreds place, etc. Find the product of 3,514 and 3.

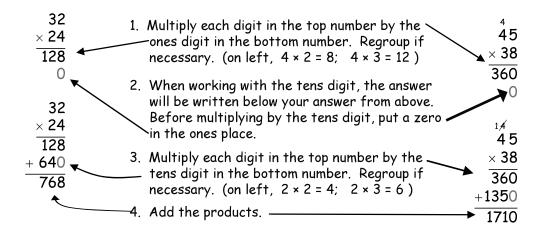
Find the product of 3,514 and 3.

$$4 \times 3 = 12$$
 ones or 1 ten and 2 ones.  
 $3,514$   
 $\times 3$   
 $1 \times 3 = 3$  tens + 1 ten (regrouped) or 4 tens.  
 $5 \times 3 = 15$  hundreds or 1 thousand and 5 hundreds.  
 $3 \times 3 = 9$  thousands + 1 thousand (regrouped) or 10 thousands.

The process for multiplying by two-digit numbers is a lot like the process above. There are a few differences. Follow the steps and examples carefully.

Examples: Multiply 32 by 24.

Find the product of 45 and 38.

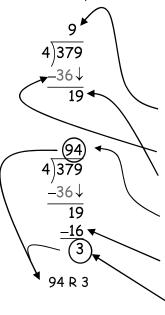


#### Solved Examples

#### Whole Numbers (continued)

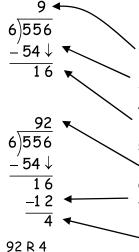
The next group of examples involves division using one-digit divisors with remainders. You already know how to divide single-digit numbers. This process, called "long division," helps you to be able to divide numbers with multiple digits.

Example: Divide 379 by 4.



- 1. In this problem, 379 is the dividend and 4 is the divisor. You're going to look at each digit in the dividend, starting on the left.
- 2. Ask yourself if the divisor (4) goes into the left-most digit in the dividend (3). It doesn't, so keep going to the right.
- 3. Does the divisor (4) go into the two left-most digits (37)? It does. How many times does 4 go into 37? (9 times)
- 4. Multiply  $4 \times 9$  (product = 36).
- 5. Subtract 36 from 37 (difference = 1). Bring down the 9 ones from the first line. This leaves 19 left from the original 379.
- 6. Ask yourself if the divisor (4) goes into 19. It does. How many times does 4 go into 19? (4 times)
- 7. Multiply  $4 \times 4$  (product = 16).
- 8. Subtract 16 from 19 (result is 3). There's nothing left to bring down from above. Once this number is smaller than the divisor, it is called the remainder and the problem is finished. The remainder is 3.
- 9. Write the answer with the remainder. (94 R 3)

**Example:** What is 556 divided by 6?



- 1. Ask yourself if the divisor (6) goes into the left-most digit in the dividend (5). It doesn't, so keep going to the right.
- 2. Does the divisor (6) go into the two left-most digits (55)? It does. How many times does 6 go into 55? (9 times)
- 3. Multiply  $6 \times 9$  (product is 54).
- 4. Subtract 54 from 55. (1) Bring down the 6 ones from the first line. This leaves 16 left from the original 556.
- 5. Ask yourself if the divisor (6) goes into 16. It does. How many times does 6 go into 16? (2)
- 6. Multiply 6 × 2 (product is 12).
- 7. Subtract 12 from 16 (result is 4). There's nothing left to bring down from above. Once this number is smaller than the divisor, it is called the remainder and the problem is finished. The remainder is 4.
- 8. Write the answer with the remainder. (92 R 4)

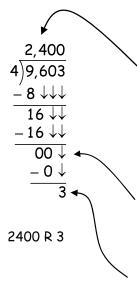
Remember: The remainder can NEVER be larger than the divisor!

#### Solved Examples

#### Whole Numbers (continued)

In some division problems there is a zero in the dividend. This process is similar to the one you used on page 290.

Example: Divide 9,603 by 4.



- 1. In this problem, 9,603 is the dividend and 4 is the divisor. Notice that there is a zero in the dividend (9,603).
- 2. Ask yourself if the divisor (4) goes into the left-most digit in the dividend (9). It does. How many times does 4 go into 9? (2 times)
- 3. Multiply  $4 \times 2$  (product = 8).
- 4. Subtract 8 from 9. (1) Bring down the 6 hundreds from above.
- 5. Look at the 16. Does the divisor (4) go into 16? It does. How many times does 4 go into 16? (4 times)
- 6. Multiply  $4 \times 4$  (product = 16).
- 7. Subtract 16 from 16. (0) Bring down the 0 tens from the first line.
- 8. Ask yourself if the divisor (4) goes into 0. It doesn't. Bring down the 3 ones from the first line.
- 9. Look at the 3. Does the divisor (4) go into 3? It doesn't. There's nothing left to bring down from above. Since this number is smaller than the divisor, the problem is finished. The remainder is 3.
- 10. Write the answer with the remainder. (2,400 R 3)

These examples involve division using two-digit divisors with remainders. Your approach will be the same as in problems with one-digit divisors.

**Example:** Divide 87 by 20.

- $\frac{4}{20)87}$   $\frac{-80}{7}$  487
- 1. Ask yourself if the divisor (20) goes into the left-most digit in the dividend (8). It doesn't, so keep going to the right.
- 2. Does the divisor (20) go into the two left-most digits (87)? It does. How many times does 20 go into 87? (4 times)
- 3. Multiply  $4 \times 20$  (product = 80).
- 4. Subtract 80 from 87 (difference = 7). There's nothing left to bring down from above. Since this number is smaller than the divisor, the problem is finished. The remainder is 7.
- 5. Write the answer with the remainder. (4 R 7)

**Example:** What is 568 divided by 11?

- $\begin{array}{r}
   51 \\
   11 \overline{\smash{\big)}568} \\
   -55 \downarrow \\
   \hline
   18 \\
   -11 \\
   7 \\
   51 R 7
   \end{array}$
- 1. Does the divisor go into the left-most digit in the dividend? No.
- 2. Does the divisor go into the two left-most digits? It does. How many times does 11 go into 56? (5 times)
- 3. Multiply  $11 \times 5$  (product = 55). Subtract 55 from 56 (result = 1).
- 4. Bring down the 8 ones from the first line. How many times does 11 go into 18? (1 time)
- 5. Subtract 11 from 18 (difference = 7). This is the remainder.

## Solved Examples

#### **Fractions**

Changing from an improper fraction to a mixed number.

**Example:** Change the improper fraction  $\frac{5}{2}$  to a mixed number.

$$\frac{5}{2}$$
 (five-halves) means  $5 \div 2$  .

So, 
$$\frac{5}{2}$$
 is equal to 2 wholes and 1 half or 2  $\frac{1}{2}$ .

Changing from a mixed number to an improper fraction.

Example: Change the mixed number  $7\frac{1}{4}$  to an improper fraction.

- You're going to make a new fraction. To find the numerator of the new fraction, multiply the whole number by the denominator, and add the numerator.
- 2. Keep the same denominator in your new fraction as you had in the mixed number.

$$7\frac{1}{4}$$
 7 x 4 = 28 28 + 1 = 29.

The new numerator is 29. Keep the same denominator, 4.

The new fraction is  $\frac{29}{4}$ .  $7\frac{1}{4}$  is equal to  $\frac{29}{4}$ .

$$7\frac{1}{4}$$
 is equal to  $\frac{29}{4}$ .

Equivalent Fractions are 2 fractions that are equal to each other. Usually you will be finding a missing numerator or denominator.

**Example:** Find a fraction that is equivalent to  $\frac{4}{5}$  and has a denominator of 35.

$$\frac{\cancel{x}}{\cancel{5}} = \frac{\cancel{?}}{\cancel{35}}$$

- 1. Ask yourself, "What did I do to 5 to get 35?" (Multiplied by 7.)
- 2. Whatever you did in the denominator, you also must do in the numerator.  $4 \times 7 = 28$  The missing numerator is 28.

So, 
$$\frac{4}{5}$$
 is equivalent to  $\frac{28}{35}$ .

**Example:** Find a fraction that is equivalent to  $\frac{4}{5}$  and has a numerator of 24.

$$\frac{\cancel{4}}{\cancel{5}} = \frac{\cancel{24}}{\cancel{2}}$$

- 1. Ask yourself, "What did I do to 4 to get 24?" (Multiplied by 6.)
- 2. Whatever you did in the numerator, you also must do in the denominator.  $5 \times 6 = 30$  The missing denominator is 30.

So, 
$$\frac{4}{5}$$
 is equivalent to  $\frac{24}{30}$ .

## Solved Examples

#### Fractions (continued)

To add (or subtract) fractions with the same denominator, simply add (or subtract) the numerators, keeping the same denominator.

Examples:

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

$$\frac{8}{9} - \frac{1}{9} = \frac{7}{6}$$

To add mixed numbers, follow a process similar to the one you used with fractions. If the sum is an improper fraction, be sure to simplify it.

$$2\frac{6}{5} \text{ is improper. } \frac{6}{5} \text{ can}$$
 be rewritten as  $1\frac{1}{5}$ .

$$S_{0}$$
,  $2\frac{6}{5}$  is  $2 + 1\frac{1}{5} = 3\frac{1}{5}$ .

#### Decimals

Adding and subtracting decimals is very similar to adding or subtracting whole numbers. The main difference is that you have to line-up the decimal points in the numbers before you begin. Add zeros if necessary, so that all of the numbers have the same number of digits after the decimal point. Before you subtract, remember to check to see if you must regroup. When you're finished adding (or subtracting), bring the decimal straight down into your answer.

Examples: Find the sum of 4.25 and 2.31.

4.25

+ 2.31

1. Line up the decimal points. Add zeroes as needed. -

- 2. Add (or subtract) the decimals.
- 3. Add (or subtract) the whole numbers.
- 4. Bring the decimal point straight down.

Add 55.2 and 6.472.

55,200 +6.47261.672

Examples: Subtract 4.8 from 7.4.

Find the difference of 4.1 and 2.88.

3 11 10 **A** . **1** Ø -2.881.22

#### Geometry

The **perimeter** of a polygon is the distance around the outside of the figure. To find the perimeter, add the lengths of the sides of the figure. Be sure to label your answer.

Perimeter = sum of the sides

**Example:** Find the perimeter of the rectangle below.

5 cm 5 cm 9 cm

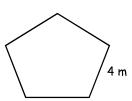
Perimeter = 5 cm + 9 cm + 5 cm + 9 cm

Perimeter = 28 cm

## Solved Examples

#### Geometry (continued)

**Example**: Find the perimeter of the regular pentagon below.



A pentagon has 5 sides. Each of the sides is 4 m long.

$$P = 4 m + 4 m + 4 m + 4 m + 4 m$$

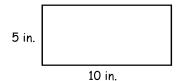
$$P = 5 \times 4 \text{ m}$$

$$P = 20 \text{ m}$$

Area is the size of a surface. To find the area of a rectangle or a square, multiply the length by the width. The area is expressed in square units ( $ft^2$ , in.<sup>2</sup>, etc.).

Area of rectangle = length  $\times$  width or  $A = L \times W$ 

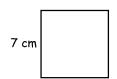
Examples: Find the area of the figures below.



Area = Length × Width

$$A = 10 \text{ in.} \times 5 \text{ in.}$$

$$A = 50 \text{ in.}^2 \implies \text{Say "50 square inches."}$$



A square has 4 equal sides, so its length and its width are the same.

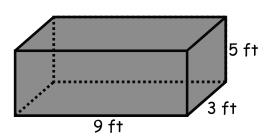
$$A = 7 \text{ cm} \times 7 \text{ cm}$$

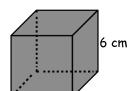
$$A = 49 \text{ cm}^2$$

Volume is the measure of space inside of a solid figure. The **volume of a rectangular** prism is the product of its length, its width, and its height. Volume of a solid is expressed in cubic units  $(m^3, ft^3, etc.)$ .

or 
$$V = L \times W \times H$$

**Examples:** Find the volume of the solids below.





 $Volume = Length \times Width \times Height$ 

$$V = 9 \text{ ft} \times 3 \text{ ft} \times 5 \text{ ft}$$

A cube has all sides equal, so its length, width, and height are all the same.

$$V = 6 \text{ cm} \times 6 \text{ cm} \times 6 \text{ cm}$$

$$V = 216 \text{ cm}^3$$

## Who Knows???

Seconds in a minute?(60)	Closed figures made up of line segments?(polygons)					
Minutes in an hour?(60)	Distance through the center of a					
Days in a year?(365)	circle?(diameter					
Months in a year?(12)	Half the diameter of a circle?(radius)					
Inches in a foot?(12)	A five-sided figure?(pentagon)					
Feet in a yard?(3)	A six-sided figure?(hexagon)					
Inches in a yard?(36)	An eight-sided figure?(octagon)					
Quarts in a gallon?(4)	A four-sided figure?(quadrilateral)					
Ounces in a pound?(16)	The answer to an addition problem?					
Pounds in a ton?(2000)	The answer to a subtraction problem?					
Cups in a pint?(2)	(difference)					
Pints in a quart?(2)	The answer to a multiplication problem?(product)					
Centimeters in a meter?(100)	The answer to a division problem?					
Grams in a kilogram?(1000)	(quotient					
Millimeters in a meter?(1000)	The average of a set of numbers?(mean)					
Years in a decade?(10)	The middle number in a set of numbers?(median)					
Years in a century?(100)	The number occurring most often?					
Degrees in a right angle?(90)	(mod					
Angle less than 90°?(acute)	Largest minus smallest?(range)					
Angle greater than 90°?(obtuse)	The perimeter?(add the sides)					
Figures with the same size and shape?(congruent)	The area?(length x width)					
Figures with the same shape, but different sizes?(similar)	The volume?(length $x$ width $x$ height)					