6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Mechanics

Teacher Notes: To find the GCF and LCM, students will rely upon their fluency with basic facts. For this standard, they will use what they know about factor pairs, prime and composite numbers, and the distributive property. Begin with a review of these concepts.

Vocabulary

composite number: a number with more than two factors; for example, 10 is a composite number because it has factors of 1, 10, 2, and 5.

distributive property of multiplication: $a(b + c) = (a \times b) + (a \times c)$ The product of a factor and a sum is equal to the sum of the products. To multiply a sum, multiply each addend separately; then add the products.

factor: a whole number that divides into another whole number; for example, 6 is a factor of 54 because 6 divides evenly into 54 (9 times).

greatest common factor: the largest factor that two or more numbers share

least common multiple: the smallest multiple that two or more numbers share

multiple: the product of a whole number and another whole number; any number divides evenly into any of its multiples.

prime factorization: a number written as the product of its prime factors; for example, $30 = 2 \times 3 \times 5$.

prime number: a number that has exactly two factors: 1 and itself

relatively prime: two numbers whose only common factor is 1; for example, 4 and 15 are relatively prime.

Part 1: Find the greatest common factor of two whole numbers that are less than or equal to 100.

Students can be taught to list a number's factors in pairs to find the GCF of the numbers.

List the factors of 12 and the factors of 30.

12	30
1, 12	1, 30
2, ⑥	2, 15
3, 4	3, 10
	5, ⑥

What is the **greatest common factor** of 12 and 30? Answer: 6

Using Prime Factorization

*Any composite number can be written as a product of its prime numbers. This is called **prime factorization**. One way to find the prime factorization of a number is to make a factor tree.*

Start with any two factors of a number. If a factor is prime, circle it. If it isn't prime, list two factors of the number. Continue until all the factors are prime. In the example below, 6 is not prime, so it isn't circled; 6 has factors of 2 and 3.

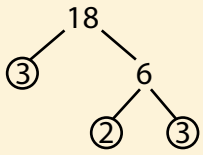
List the prime factors from least to greatest, and use exponents to shorten the final answer.

Examples

Find the prime factorization of 18.

Answer: $18 = 2 \times 3 \times 3$

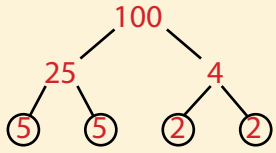
$18 = 2 \times 3^2$



Find the prime factorization of 100.

Answer: $100 = 2 \times 2 \times 5 \times 5$

$100 = 2^2 \times 5^2$



Use prime factorization to find the GCF of 12 and 30.

List the prime factors of 12. Answer: **2** × 2 × **3**

List the prime factors of 30. Answer: **2** × **3** × 5

What are the common prime factors of 12 and 30? The product of these two numbers (2 × 3) is the greatest common factor of 12 and 30. Answer: The GCF is 6.

Use the GCF and the distributive property to rewrite 12 + 24. 12(1 + 2)

Use the GCF and the distributive property to rewrite 35 + 49. 7(5 + 7)

Use the GCF and the distributive property to rewrite 64 + 56. 8(8 + 7)

Use the GCF and the distributive property to rewrite 40 + 50. 10(4 + 5)

Use the GCF and the distributive property to rewrite 25 + 35. 5(5 + 7)

Being able to find the GCF can help you to solve real-world problems. Look for the greatest number of items that can be used.

Examples

Jenny is making treat bags. She has 12 balloons and 16 pieces of candy. All the bags will be the same and no items will be left over. What is the greatest number of treat bags that she can make? How many of each item will go in the bags?
Answer: The GCF is 4, so Jenny can make 4 bags with 3 balloons and 4 pieces of candy in each bag.

Marcus is babysitting this summer. He charges the same amount for each session, and he works no more than 10 sessions per week. During the first week, he makes $35, and during the second week, he makes $56; during the third week, he makes $70. What is the highest fee Marcus could be charging for babysitting?
Answer: The GCF of 35, 56, and 70 is 7. The most Marcus could be charging is $7 per session.

Christopher has 7 tulips and 14 sunflowers to make flower arrangements. If Christopher wants all of the arrangements to have the same number of flowers, what is the greatest number of arrangements he can make?
Answer: The GCF is 7, so there will be 7 arrangements with 1 tulip and 2 sunflowers in each.

Austin is decorating trees. He has 18 red streamers, 9 strings of lights, and 27 ornaments. Each tree needs to have the same number of items. At most, how many trees can he decorate?
Answer: The GCF is 9, so 9 trees can be decorated, each with 2 red streamers, 1 string of lights, and 3 ornaments.

Part 2: Find the least common multiple of two whole numbers that are less than or equal to 12.

Students can be taught to list multiples of each number until they come across a common multiple.

What is the least common multiple of 6 and 7? List the multiples of each until you find it.

6: 6, 12, 18, 24, 30, 36, **42**…
7: 7, 14, 21, 28, 35, **42**… Answer: 42

What is the least common multiple of 3 and 17? List the multiples of each until you find it.

3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, **51**, 54, 57…
17: 17, 34, **51**, 68… Answer: 51

Another way to find the LCM is by using a "ladder."

Find the LCM of 4 and 10.

2 | 4, 10
2 | 2, 5
5 | 1, 5
1, 1

Inside the ladder, write both numbers.
On the outside, write a factor of both.
Continue dividing until the only factor left is 1.
Multiply the numbers to the left of the ladder.

$2 \times 2 \times 5 = 20$ The LCM of 4 and 10 is 20.

Shopping For the Largest Unique Primes

*Students can be taught to use prime factorization to find the LCM of two numbers. After finding the prime factorization of each number in the set, students will be asked to circle the **largest unique prime** number.*

To aid in this process, review prime numbers and create a list of the first 10 or so primes (2, 3, 5, 7, 11, 13, 17, 19, 23, and 29). Refer to this list when determining the largest unique prime.

Students can then think of this process as "Shopping for Primes."

Find the LCM of 8 and 6.

First, find the prime factorization of each number and write it in exponential form.

$8 = 2 \times 2 \times 2$ $8 = 2^3$
$6 = 2 \times 3$ $6 = 2 \times 3$

Now, go shopping. You want "one of everything" (**unique**) and the **largest**. In other words, you will choose one of every prime represented, but make sure it is the one with the largest exponent.

These are the primes you're looking for: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and so on.

$8 = (2^3)$ Are there any 2s? Yes, a 2 and a 2^3. Circle 2^3 because it's the largest.

$6 = 2 \times (3)$ Are there any 3s? Yes, circle it. Are there any 5s, any 7s, etc.?

No, there are no other unique prime numbers.

Multiply the circled factors; they are the **largest unique prime** factors of 8 and 6.
Answer: $2^3 \times 3 = 24$; the LCM of 6 and 8 is 24

Find the LCM of three numbers: 12, 24, and 15.

$12 = 2^2 \times 3$
$24 = 2^3 \times 3$
$15 = 3 \times 5$

Start by finding the prime factorization of all three numbers. Use exponential notation.

(Study the entire group of factors for all three numbers.)

$12 = 2^2 \times (3)$
$24 = (2^3) \times 3$
$15 = 3 \times (5)$

I want one of every prime, and I want the largest one. The primes in this group are 2, 3, and 5. Circle one of each.

I chose 2^3 because it has the largest exponent, but I can circle any one of the 3s, and there is only one 5, so that gets circled.

Multiply the largest unique primes.

Answer: The product of 2^3 and 3 and 5 is 120, and that is the LCM of 12, 24, and 15.

Find the LCM of 4 and 15.

First, find the prime factorization of each number and write each in exponential form.

$4 = 2 \times 2 = 2^2$
$15 = 3 \times 5$

Then, circle the largest of each unique prime.

$4 = (2^2)$
$15 = (3) \times (5)$

Finally, multiply the circled factors representing the highest of each unique prime factor.
Answer: $2^2 \times 3 \times 5 = 60$; the LCM of 4 and 15 is 60

Notice that these numbers are **relatively prime** (their only common factor is 1). In cases such as this, the LCM is the product of the two numbers: 4×15 or 60.

More Practice Examples

Find the LCM of 25 and 20.

First, find the prime factorization of each number and write each in exponential form.

$25 = 5 \times 5 = 5^2$
$20 = 2 \times 2 \times 5 = 2^2 \times 5$

Then, circle the largest of each unique prime.

$25 = (5^2)$
$20 = (2^2) \times 5$

Finally, multiply the circled factors representing the highest of each unique prime factor.
Answer: $2^2 \times 5^2 = 100$; the LCM of 25 and 20 is 100.

Find the LCM of 3 and 7.

When both numbers are prime numbers, the LCM is the product of the two numbers.
Answer: $3 \times 7 = 21$; the LCM of 3 and 7 is 21.

Find the LCM of 8 and 4. 8

Find the LCM of 9 and 12. 36

Find the LCM of 3 and 4. 12

Find the LCM of 24 and 36. 72

Find the LCM of 12, 15, and 20. 60

LCM can be found in real world problems. These problems often ask when an event will happen again or what will be the smallest number in a group.

Examples

The red bus arrives at the bus stop every 10 minutes. The blue bus arrives every 5 minutes. Both are at the stop right now. In how many minutes will both be at the bus stop again?

Answer: $5 \times 2 = 10$
They will both arrive together in 10 minutes.

(5) | 10, 5
(2) | 2, 1
| 1, 1

Heather is training for a biathlon. She bikes on every 12th day and swims on every 6th day. Heather did both activities today. How many days from now will she do both again?
Answer: The LCM is 12, so in 12 days she will do both again.

The Super Saver sells hot dogs in packages of 12 and buns in packages of 8. If you want to purchase the same number of each, what is the minimum number of packages of buns and packages of hotdogs would you have to buy?
Answer: The LCM is 24, so you would need to buy 24 buns (2 packages) and 24 hotdogs (3 packages).

Ryan is thinking of a number that is divisible by both 11 and 5. What is the smallest possible number that Ryan could be thinking of?
Answer: 55 because the LCM of 11 and 5 is 55.

Jenny goes to dance class every 4 days. McKayla goes every 6 days. They were both in class together today. In how many days will they both be at dance again?
Answer: They will both be at class together in 12 days; the LCM of 4 and 6 is 12.

Part 3: Use the distributive property to express a sum of two whole numbers between 1 and 100 with a common factor as a multiple of a sum of two whole numbers with no common factor.*

*This can only be done if there is a common factor. If there is no common factor, write "no common factor." For example, there is no common factor for 19 and 63.

Use the GCF and the distributive property to rewrite 9 + 15.

$9 = 3 \times 3$ $\quad$ $15 = 3 \times 5$ $\quad$ the GCF of 9 and 15

$3(3 + 5)$

These two have no common factor.

This answer can be checked:

$9 + 15 = 3(3 + 5)$

$24 = 3 \times 8$

$24 = 24$ ✓

Use GCF and the distributive property to rewrite 24 + 36.

$24 = 2 \times (2 \times 2 \times 3)$

$36 = (2 \times 2 \times 3) \times 3$

$12(2 + 3)$

Find the GCF of the numbers. This number goes outside the parentheses. Notice the GCF is 12 because $2 \times 2 \times 3$ is 12.

The remaining numbers become addends and go inside the parentheses: 2 and 3 are the remaining numbers.

Study the examples. Use GCF and the distributive property to rewrite 32 + 24.

32	24
1, 32	1, 24
2, 16	2, 12
4, 8	3, 8
	4, 6

The GCF of 32 and 24 is 8. $32 + 24 = 8(4 + 3)$

Study the example. Rewrite each equation and simplify both sides.

$39 + 30 = 3(13 + 10)$
$69 = 3 \times 23$
$69 = 69$

$42 + 48 = 6(7 + 8)$
$90 = 6 \times 15$
$90 = 90$

$24 + 40 = 8(3 + 5)$
$64 = 8 \times 8$
$64 = 64$

$36 + 27 = 9(4 + 3)$
$63 = 9 \times 7$
$63 = 63$

$28 + 21 = 7(4 + 3)$
$49 = 7 \times 7$
$49 = 49$

$55 + 66 = 11(5 + 6)$
$121 = 11 \times 11$
$121 = 121$

Concept Mastery

- ✓ Students are able to find the greatest common factor of two whole numbers less than or equal to 100.
- ✓ Students are able to find the least common multiple of two whole numbers less than or equal to 12.
- ✓ Students are able to use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.

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Models for Multiples and Factors

Class Demo & Activities

Finding Factors of a Number

Prime Factor Finder

Prime and Composite Numbers

Multiples, Factors, and Primes *A variety of tutorials and teacher resources are available.*

Prime and Composite Numbers Practice

Prime and Composite Lesson & Practice

Student Practice

Finding Prime Factors *Great tutorial, but when doing the exercises at the end, it does take a moment before the next button illuminates.*

Greatest Common Factors

Least Common Multiples

Factor Trees, Prime Factorization, GCF, and LCM

Factorization Forest

Giant Frozen Rubber Turkeys of Destruction Prime Factoring

Prime Factorization Mini Lesson with Student Quiz

Prime Factorization *Includes answers.*

Prime Numbers *Includes answers.*

Prime Number Shooter

Prime Factorization Mini Lesson with Student Problems

Prime Factorization Online Quiz

Factor Game

Factors and Multiples Jeopardy Game

Factors Millionaire Game

LCM and GCF Word Problems

Least Common Multiple

Prime Numbers

Prime Landing Mission

Factor Feeder

Number Cop

King Kong's Prime Numbers

Sliding Factors

Pop the Prime Number

Least Common Multiple Mini Lesson with Student Practice

Least Common Multiple by Factor Trees with Student Practice

Least Common Multiple Match

Greatest Common Factor Match

Prime Factoring Mini Lesson with Student Practice

Prime Factorization with Student Practice

Divisibility Rules

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Factor Bingo

Greatest Common Factor Game

Least Common Denominator Fruit Shoot

Greatest Common Factor Fruit Shoot

Prime/Composite Numbers Fruit Shoot

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References

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