8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
For example, $3^2 \times 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$.

Mechanics

Teacher Notes: This standard follows from 6.EE.1 and 6.EE.2.c. There are no Level 7 standards which directly address exponents. Because of this gap, some review of exponents is recommended before introducing this standard.

In addition, students often have greater success applying exponents to variables than they do applying them to integers. A common misconception is to multiply the exponent by the base. Review of exponents and how they're applied to integers will facilitate success with this standard.

This standard covers the following properties involving operations with exponents.

- *Addition and subtraction of terms*
- *Product and quotient rules*
- *Raising a product and a quotient to a power*
- *Raising a power to a power*
- *Raising to a power of 0*
- *Negative exponents*

Addition and Subtraction of Terms

In order to add or subtract exponential terms, they must have both the same base as well as the same exponent. (Only the coefficient can be different.) Terms that have both the same base and the same power are called like terms. The addition and/or subtraction of like terms is commonly referred to as "combining like terms."

Simplify $2a^2 - 3a + 5a$.

This expression contains 3 terms: $2a^2$, $-3a$, and $+5a$. Of these, two are like terms, $-3a$, and $+5a$, which can be combined; the "unlike" terms can't be changed and are left as is.

When like terms are combined, the base and the power are kept the same and the coefficients are added (or subtracted).

$-3a$, and $+5a = +2a$

So, overall, the original expression can be simplified to $2a^2 + 2a$.

There is no limit to the number or type of terms that can be combined in this way.

Examples:

Simplify $7 - x - 3 + 4x$.

$7 - 3 - x + 4x$

$4 + 3x$

The first step is to group like terms together.

Remember: Constants (integers without a variable) are also like terms.

Simplify $2x^2 + 3y + x^2 - 2y + 3y^2$.

$2x^2 + x^2 + 3y - 2y + 3y^2$

$3x^2 + y + 3y^2$

Remind students that if a variable is written without a coefficient, it is the same as having a coefficient of 1. That is, $x^2 = 1x^2$.

Product of Powers

Find $y^2 \times y^4$.

It is easy to understand this problem if you expand it first. $y^2 = y \times y$; $y^4 = y \times y \times y \times y$. So now you have the following:

$y^2 \times y^4 = (y \times y) \times (y \times y \times y \times y) = y \times y \times y \times y \times y \times y = y^6$

$y^2 \times y^4 = y^6$

This process could be very time consuming. However, if we notice the relationship between the exponents that are being multiplied together and the exponent of the product, we can avoid writing it out each time.

By definition, $a^n \times a^m = a^{(n+m)}$.

$y^2 \times y^4 = y^{2+4} = y^6$

So, when multiplying exponents, simply add the exponents and keep the base. It is extremely important that students understand this — you can only add the exponents when they have the same base. (If the problem were $x^2 \times y^4$, there are two different bases, x and y, so they could not be simplified this way.)

There are other equivalent ways to show that exponents are being multiplied, e.g. $(y^2)(y^4) = y^{(2+4)} = y^6$.

Find $a^{-5} \times a^3$.

In this example, one of the exponents is negative. The same rules apply.

$a^{-5} \times a^3 = a^{(-5+3)} = a^{-2} = 1/a^2$

Quotient of Powers

Now, let's talk about dividing with exponents.

Find $y^2 \div y^4$.

Remember that a fraction is another way to write a division problem. Write it as a fraction and then expand it.

$$\frac{y^2}{y^4} = \frac{y \times y}{y \times y \times y \times y}$$

Then, begin canceling out *y*s until either the numerator or denominator runs out.

$$\frac{\cancel{y} \times \cancel{y}}{\cancel{y} \times \cancel{y} \times y \times y} = \frac{1}{y \times y} = \frac{1}{y^2} = y^{-2}$$

By definition, $a^n \div a^m = a^{(n-m)}$.

In other words, $y^2 \div y^4 = y^{(2-4)} = y^{-2}$. Notice the relationship between the exponents of the dividend and divisor and the exponent of the quotient. 2 – 4 = –2.

$$y^2 \div y^4 = y^{2-4} = y^{-2}$$

Therefore, whenever you divide numbers with exponents, you keep the base and subtract the exponents. Just as we saw with multiplication, this is only true when the exponents have the same base. (In the example above, the base is *y*.)

Remind student that there are other ways to show division such that exponents are being subtracted (e.g. $\frac{x^5}{x^2} = x^{(5-2)} = x^3$).

Raising a Product or a Quotient to a Power $(ab)^x = a^x \times b^x$ or $(\frac{a}{b})^x = \frac{a^x}{b^x}$

When a product is raised to a power, the exponent is applied to each factor.

Simplify $(ab)^2$.
$(ab)^2 = a^2 \times b^2$

Simplify $(3x)^3$.
$(3x)^3 = 3^3 \times x^3 = 27x^3$

In a similar way, a power is applied to a quotient. When a quotient is raised to a power, the exponent is applied to both the dividend and the divisor.

Simplify $(\frac{a}{b})^2$. $\quad (\frac{a}{b})^2 = \frac{a^2}{b^2}$

Simplify $(\frac{3x}{2})^4$. $\quad \frac{(3x)^4}{2^4} = \frac{3^4 \times x^4}{16} = \frac{81x^4}{16}$

The rule is $(\frac{a}{b})^n = \frac{a^n}{b^n}$. *(It is important for students to distinguish between $(\frac{a}{b})^n$ and $\frac{a^n}{b}$ and $\frac{a}{b^n}$. Confusion of these often leads to errors. There must be parentheses if the entire fraction is raised to a power.)*

Raising a Power to a Power $(a^x)^y = a^{xy}$

Find $(y^2)^4$.

It is easy to understand this problem if you expand it first.
$(y^2)^4 = (y \times y)(y \times y)(y \times y)(y \times y) = y \times y \times y \times y \times y \times y \times y \times y = y^8$

So now you have the following:

$(y^2)^4 = y^8$

Again, this could become very time consuming. However, if we notice the relationship between the exponent that is raised to a power, we can avoid writing it out each time.

By definition, $(a^x)^y = a^{(xy)}$. When a power is raised to a power, keep the base and multiply the exponents.

Find $(2y^2)^5$.

$(2y^2)^5 = 2^5 y^{(2\times5)} = 32y^{10}$

Find $((2y)^2)^5$.

$((2y)^2)^5 = (2y)^{2\times5} = (2y)^{10} = 2^{10} \times y^{10} = 1{,}024y^{10}$

Raising to a Power of Zero $a^0 = 1$

This is a rule that is simple to memorize but can be explained by applying the quotient of powers rules. According to the quotient of powers rule:

$$\frac{a^5}{a^5} = a^{5-5} = a^0$$

We also know that any number divided by itself is equal to 1.

$$\frac{a^5}{a^5} = 1$$

Therefore, $\frac{a^5}{a^5} = 1$.

Find $(2y)^0 = 1$

Find $(3x^2 + y + 3y^2)^0 = 1$

Find $658^0 = 1$

(Be careful of parentheses, however!) $2y^0 = 2 \times y^0 = 2 \times 1 = 2$

Negative Exponents $a^{-b} = \frac{1}{a^b}$ *and* $a^b = \frac{1}{a^{-b}}$

Negative exponents are are extension of the quotient rule.

If we look at $\frac{y^2}{y^6}$, we see that $\frac{y^2}{y^6} = y^{2-6} = y^{-4}$.

Another way to look at this is by expanding the fraction.

$$\frac{y^2}{y^6} = \frac{\cancel{y} \times \cancel{y}}{\cancel{y} \times \cancel{y} \times y \times y \times y \times y} = \frac{1}{y \times y \times y \times y} = \frac{1}{y^4}$$

This tells us that $y^{-4} = \frac{1}{y^4}$

Find $\left(\frac{1}{3}\right)^{-2}$. $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^{2} = \frac{3^2}{1^2} = 9$

Find 3^{-3}. $3^{-3} = \frac{3^{-3}}{1} = \frac{1}{3^3} = \frac{1}{3 \times 3 \times 3} = \frac{1}{27}$

Find $\left(\frac{2}{5}\right)^{-3}$. $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3} = \frac{5^3}{2^3} = \frac{125}{8}$

***Simple Solutions Common Core Math 8* Examples**

Simplify. $\frac{p^8q^4}{p^2q^5}$ — $\frac{p^6}{q}$ *or* p^6q^{-1}

Simplify using exponential notation: $3b^3 \times 5b^4$ — $15b^7$

Simplify using exponential notation: $12^3 \times 12^3 \times 12^2$ — 12^8

Write using only positive exponents. $\frac{f^{-8}}{f^{-3}}$ — $\frac{f^3}{f^8} = \frac{1}{f^5}$

Simplify using only positive exponents. $(4p)^{-3}$ — $\frac{1}{64p^3}$

Simplify. $\frac{5^8x^6}{5^5x^7}$ — $\frac{5^3}{x} = \frac{125}{x}$ or $125x^{-1}$

Finally, students will need to solve problems that mix numerals and variables. Instruct students that like bases are multiplied (i.e. numerals to numerals and matching variables to matching variables, such as *y* to *y*).

Simplify $4y^2 \times 6y^5$.

Group like terms together. $4y^2 \times 6y^5 = 4(6) \times (y^2)(y^5) = 24y^{(2+5)}$ $24y^7$

Simplify $\frac{10x^5y^3}{4x^2y^4}$ using exponential notation. $\frac{5x^3}{2y}$ or $\frac{5}{2}x^3y^{-1}$

Simplify $\frac{7^{21}m^6}{7^4m^{40}}$. $\frac{7^{17}}{m^{34}}$ or $7^{17}m^{-34}$

Concept Mastery

- ✓ Students are able to apply the properties of operations involving both positive and negative exponents modifying the same base. (The base in this case might be a variable or a numeral.)
- ✓ Students are able to apply the properties of operations involving both positive and negative exponents in problems involving multiple bases. (The bases in this case might be all variables, all numerals, or a mixture.)

A link to helpful web resources can be found on page 146 of the full Level 8 document.

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