7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Mechanics

Teacher Notes: In this standard we will solve proportion-related problems using several different methods. Our first step will be to look at the justification for cross-multiplication.
In earlier grade levels, students have used ratio tables and unit rates to solve proportion problems. In grade 7, students can begin solving proportions through common denominators. Use informal reasoning to guide the students. Once this method is clear, the algorithm of cross-multiplication can be applied.

Frankie is throwing a party. Originally, Frankie was expecting 16 guests and ordered 18 liters of cola. At the last minute, Frankie found out that 8 more guests were coming. How many total liters of cola will Frankie need?

$$\frac{18}{16} = \frac{x}{24}$$

$$\frac{24}{24}\left(\frac{18}{16}\right) = \frac{x}{24}\left(\frac{16}{16}\right) \qquad \text{Note: } \frac{24}{24} \text{ and } \frac{16}{16} = 1$$

$$\frac{24}{24}\left(\frac{18}{16}\right) = \frac{x}{24}\left(\frac{16}{16}\right) \qquad 24(16) = (24)16$$

So, $24(18) = x(16)$

$$x = \frac{24(18)}{(16)} = \frac{432}{16} = 27$$

- Identify the known ratio. Identify the unknown. *(Frankie needs 18 liters of cola for 16 guests. We want to know how many liters are needed for 24 guests.)*
- Set up a proportion.
- Multiply each ratio times a ratio $\frac{x}{x}$, where x represents the denominator from the opposite ratio.
- Note that the denominators for both ratios are now the same. If the denominators are the same, the numerators are equal.
- Drop the denominators and solve for x.

Frankie will need 27 liters of cola.

Reinforce the meaning of x = 27. Ask the students to explain using a sentence. "If 18 liters of cola are needed for 16 guests, then 27 liters of soda are needed for 24 guests.

Notes:

The first step is getting students to see that if the denominators are equal, then the numerators will also be equal. Then build on previous knowledge by pointing out that students know how to get common denominators.

Explain that multiplying each denominator by the other is one way to find a common denominator. Show that doing so in the form $\frac{x}{x}$ is the same as multiplying by 1, therefore preserving the value of the denominator.

Circle the two denominators to show that they are equal. Explain why the denominators can be dropped.

$$\frac{a}{b} = \frac{c}{d}$$

$$\text{If } b = d, \text{then } a = c$$

Once the students have a solid understanding of the common denominator method, introduce the cross-multiplication shortcut. Here, the cross-multiplication shortcut is applied to the previous example.

To cross-multiply:

1. Multiply downward on each diagonal.
2. Set the products of the diagonals equal to each other.
3. Solve for the missing variable.

Once you have walked through how to cross-multiply, connect it to the common denominator method by showing that they will lead to the same outcome.

Cross-Multiply Shortcut

$$\frac{18}{16} = \frac{x}{24}$$

$$24(18) = 16(x)$$

$$\frac{432 = 16x}{16}$$

$$x = 27$$

Common Denominator Method

$$\frac{18}{16} = \frac{x}{24}$$

$$\frac{24}{24}\left(\frac{18}{16}\right) = \frac{x}{24}\left(\frac{16}{16}\right)$$

$$24(18) = x(16)$$

$$\frac{432 = 16x}{16}$$

$$x = 27$$

Proportion problems can be solved in a variety of ways. The Common Core Standards emphasize solving proportions using intuitive reasoning, rather than a set algorithm. Here is a summary of some of the ways we have learned to think about proportions.

- *Proportions are related by the same factor between ratios.*
- *Proportions are related by the same factor within ratios.*

$$\frac{2}{5} \overset{\times 2}{\underset{\times 2}{=}} \frac{4}{10}$$

- *Proportions can be solved using common denominators/cross multiplication.*
- *Proportions follow the equation of the form* y = kx.

$$\times 2.5 \left(\frac{2}{5} = \frac{4}{10} \right) \times 2.5$$

Below is an illustration of one using several different methods.

25 people visited the dentist on Monday. 15 had no cavities. What percent of patients did not have cavities?

Method 1: $\frac{15}{25} \overset{\times 4}{\underset{\times 4}{=}} \frac{x}{100}$

Recognize that 100 is 4 times as great as 25. This means that x must be 4 times as great as 15, or 60. $\frac{15}{25} = 60\%$

Method 2: $\times 0.6 \left(\frac{15}{25} = \frac{x}{100} \right) \times 0.6$

Compare the numerator to the denominator; 15 is ³⁄₅ (or ⁶⁄₁₀) of 25. Therefore, x will be ³⁄₅ (or ⁶⁄₁₀) of 100. The ratio of 60 : 100 is the same as 60%.

Method 3:

$$\frac{15}{25} = \frac{x}{100}$$

$$\frac{100 \times 15}{100 \times 25} = \frac{x \times 25}{100 \times 25}$$

$$\frac{100 \times 15}{\cancel{100 \times 25}} = \frac{x \times 25}{\cancel{100 \times 25}}$$

$$1{,}500 = 25x$$

$$60 = x \qquad \frac{15}{25} = 60\%$$

Change both fractions to common denominators by multiplying by the opposite denominator. Drop the denominators and solve. The cross-multiplication method can also be used.

Method 4:

$$15 = k \times 25$$

$$\frac{15}{25} = k$$

$$0.6 = k$$

$$0.6 \times 100 = 60\%$$

Use the equation $y = kx$. Percent is an amount per hundred.

When solving proportion related problems, students should be able to use a variety of methods. The idea is to help students use reasoning skills and choose the situation that best fits the problem. In the examples that follow, we have included solutions using proportions as well as equations. It is up to the individual teacher to decide whether these methods should be introduced simultaneously or separately.

Apply the use of proportional relationships to ratio and percent problems involving taxes, markups and markdowns, gratuities and commissions, percent increase or decrease, percent error and simple interest.

Each type of ratio or percent will be contextualized and illustrated as a proportion.

A **tax** is a charge levied by the government. Some examples of taxes include income tax, real-estate tax, and sales tax. A tax is expressed as a percentage (e.g. a sales tax of 5.25% means that the goverment charges an additional fee that is 5.25% of the final cost of the purchase.)

Louise purchased a tennis racquet for $55.00. The sales tax was 6.75%. Write a proportion and an equation to show how to calculate the sales tax.

$$\frac{x}{55} = \frac{0.0675}{100}$$

$$y = kx$$

$$\text{tax} = 0.0675 \times 55$$

Charlie had $55.00. He wanted to buy a pair of jeans for $50. A sales tax of 7% was added to the purchase price. Did Charlie have enough money?

$$\overbrace{\underbrace{\frac{x}{50} = \frac{7}{100}}_{2}}^{2};\ x = 3.5$$

total cost: $50 + 3.5 = $53.50

Charlie had enough money.

$$y = kx$$

$$y = 0.07 \times 50$$

$$3.5 = 0.07 \times 50$$

total cost: $50 + 3.5 = $53.50

Charlie had enough money.

A **markup** is an amount added to the cost of an item in order to determine the retail price. For example, a retail store might add a markup of 25% to the cost of the shirt.

The average markup on clothing at Betty's Boutique is 56%. Write a proportion and an equation to show how to calculate the markup on a blouse that cost the boutique $30.

$$\frac{x}{30} = \frac{56}{100}$$

$$y = kx$$

markup = percentage × cost of the shirt

markup = 0.56 × 30

Joe builds and sells bookcases. The supplies for one bookcase cost $45. After applying a markup of 100%, how much does Joe charge for one bookcase?

$$1\left(\frac{x}{45} = \frac{100}{100}\right)1;\ x = \$45 \text{ markup}$$

original cost + markup = price

45 + 45 = $90

$$y \times kx$$

markup = percentage × cost of bookcase

markup = 1 × 45 =

markup = 45

45 + 45 = $90 (amount Joe charges)

A **markdown** is an amount subtracted from the original sales price.

Fred's Appliance Store is having a sale. Dishwashers, which were sold for $450, are now on sale for 25% off. Write a proportion and an equation to calculate the markdown amount.

$$\frac{x}{450} = \frac{25}{100}$$

$x = \$112.50$

$$y = kx$$

$$y = 0.25 \times 450$$

$y = \$112.50$

The day after Halloween, Buy-Rite marked all candy 50% off. If sales tax is 6.25%, how much would a $10.00 bag of candy now cost?

calculate sale price

$$\times 2\ \left(\frac{x}{10} = \frac{50}{100}\right)\times 2;\ x = \$5$$

calculate tax

$$\left(\frac{100}{100}\right)\frac{x}{5} = \frac{6.25}{100}\left(\frac{5}{5}\right)$$

$$\frac{100 \times x}{\cancel{100 \times 5}} = \frac{6.25 \times 5}{\cancel{100 \times 5}}$$

$$x = \frac{6.25 \times 5}{100} = 0.31$$

sale price + tax = total cost

$5.00 + 0.31 = $5.31

calculate sale price

$$y = kx$$

$$y = 0.50 \times 10$$

$$y = \$5.00$$

calculate tax

$$y = kx$$

$$y = 0.0625 \times 5$$

$$y = 0.31$$

sale price + tax = total cost

$5.00 + 0.31 = $5.31

The daily special at Brite Prints is \$35 off any purchase of at least \$100. Lauren has a coupon for 25% off, but she cannot combine the coupon with the daily special. If Lauren's purchase is \$115, which offer will save her more money?

<u>daily special</u>

savings = \$35.00

<u>with coupon</u>

$$\times 4\left(\frac{x}{115} = \frac{25}{100}\right)\times 4$$

$115 \div 4 = \$28.75$

savings = \$28.75

<u>daily special</u>

savings = \$35.00

<u>with coupon</u>

$$y = kx$$

$$y = 0.25 \times 115$$

$$\$28.75 = 0.25 \times 115$$

savings = \$28.75

The daily special will save Lauren more money.

Mark purchased a bike on sale. Originally, the bike cost \$250. However, the sale offered a markdown of 35%. If sales tax was 6.50%, how much did Mark pay for the bike?

Students should recognize that 35% off an item is the same as paying 65% for the item.

cost of bike on sale

$100\% - 35\% = 65\%$

$$\frac{x}{250} = \frac{65}{100};\quad 65 \times 2.5 = \$162.5$$

(2.5, 2.5)

calculating tax

$$\left(\frac{100}{100}\right)\frac{x}{162.5} = \frac{6.5}{100}\left(\frac{162.5}{162.5}\right)$$

$$\frac{100x}{\cancel{100 \times 162.5}} = \frac{6.5 \times 162.5}{\cancel{100 \times 162.5}};\ x = \frac{6.5 \times 162.5}{100}$$

$x = 10.56 = \text{tax}$

total cost = 162.50 + 10.56 = \$173.06

cost of bike on sale

$100\% - 35\% = 65\%$

$$y = kx$$

$$y = 0.65 \times 250$$

$$162.50 = 0.65 \times 250$$

calculating tax

$$y = kx$$

$$y = 0.0650 \times 162.50$$

$$10.56 = 0.0650 \times 162.50$$

total cost: 162.50 + 10.56 = \$173.06

A **gratuity** or **commission** is a fee paid to service personnel or employees in return for a service. Both are based on a percent of the sale. For example, the gratuity a waitress receives is a percentage of the cost of the meal. The commission a car salesman receives is a percentage of the sales price of the car.

The Murray family's restaurant bill came to \$87.50. Mr. Murray wanted to leave the waitress a 20% tip. Write a proportion and an equation to show how to determine the amount of the tip.

$$\frac{x}{87.50} = \frac{20}{100}$$

$x = \$17.50$ tip

$$y = kx$$

$$y = 0.20 \times 87.50$$

$y = \$17.50$ tip

Mr. and Mrs. Johnson have a "buy one, get one" coupon for dinner at a local restaurant. They order two steak dinners for $27.95 each, and the total for their drinks is $7. Mrs. Johnson wants to leave a 20% tip for the cost of the entire meal, even though she gets one dinner free. Calculate the total cost that Mrs. Johnson will pay, including the tip.

price of two meals and drink

$$2(\$27.95) + 7 = 55.90 + 7 = \$62.90$$

cost of tip

$$5\left(\frac{x}{62.90} = \frac{20}{100}\right)5$$

$$62.90 \div 5 = 12.58$$

total cost Mrs. Johnson will pay

$$\$62.90 - \$27.95 = 34.95 + 12.58 = \$47.53$$

price of two meals and drink

$$2(\$27.95) + 7 = 55.90 + 7 = \$62.90$$

cost of tip

$$y = kx$$

$$y = 0.20 \times 62.90$$

$$12.58 = 0.20 \times 62.90$$

total cost Mrs. Johnson will pay

$$\$62.90 - \$27.95 = 34.95 + 12.58 = \$47.53$$

George earns 13.75% commission on every car he sells. On Wednesday, he sold 2 cars priced at $23,234 each. How much money in commission did George earn?

$$\frac{x}{\$23{,}234} = \frac{13.75}{100}$$

$$\left(\frac{100}{100}\right)\frac{x}{\$23{,}234} = \frac{13.75}{100}\left(\frac{23{,}234}{23{,}234}\right)$$

$$\frac{100x}{\cancel{100 \times 23{,}234}} = \frac{13.75 \times 23{,}234}{\cancel{100 \times 23{,}234}}$$

$$x = \frac{13.75 \times 23{,}234}{100}$$

$x = \$3{,}194.68$ commission for one car

$\$3{,}194.68 \times 2 = \$6{,}389.35$ total commission

$$y = kx$$

$$y = (0.1375)(23{,}234)$$

$y = \$3{,}194.68$ commission for one car

$\$3{,}194.68 \times 2 = \$6{,}389.35$ total commission

In percent change and percent error problems, the difference must be calculated prior to calculating the proportion.

Percent change, which includes **percent increase** and **percent decrease**, compares the amount of change to the original price. In looking at percent change we are asking, "Did the value change a lot or a little?"

At Hank's Hamburger Hut, the cost of a hamburger increased from \$4.50 to \$5.25. Find the percent of increase.

Depending on the students' level, you can show this problem broken down into two steps, as shown above, or written together as shown below.

$$\$5.25 - \$4.50 = 0.75$$

$$\frac{0.75}{4.50} = \frac{x}{100}$$

$$\frac{0.75 \times 100}{4.50} = x$$

$$x = 16.7\% \text{ increase}$$

alternate setup

$$\frac{(5.25 - 4.50)}{4.50} = \frac{x}{100}$$

$$\$5.25 - \$4.50 = 0.75$$

$$0.75 = k \times 4.50$$

$$k = \frac{0.75}{4.50}$$

$$k = 16.7\%$$

When we calculate **percent error** we are looking at the difference between our own value and an accepted value as it compares to the accepted or true value. In looking at percent error, we are asking "Was my value close to the accepted value or far away?"

In physics class, John calculated the force of gravity as 7.6 m/s^2. The accepted value for the force of gravity (on Earth) is 9.8 m/s^2. Solve for the percent error.

$$\frac{(9.8 - 7.6)}{9.8} = \frac{x}{100}$$

$$\frac{2.2}{9.8} = \frac{x}{100}$$

$$\frac{2.2 \times 100}{9.8} = x$$

$$x = 22.45\% \text{ error}$$

$$y = kx$$

$$(9.8 - 7.6) = k \times 9.8$$

$$2.2 = k \times 9.8$$

$$k = 22.45\% \text{ error}$$

Interest is a charge paid when money is borrowed or a profit gained when money is loaned. **Simple interest** is calculated as a percentage of the **principal** (money borrowed/loaned).

$$\frac{\text{interest}}{\text{principal}} = \frac{x}{100}$$

For simple interest, the equation $y = kx$ is seen as $I = prt$, where I = interest, p = principal, r = rate and t = time. (k = principal * rate)

Dan takes a loan of \$3,000 at an annual percentage rate of 5.5% simple interest. Write a proportion and an equation to show how much interest Dan will pay per year.

$$\frac{x}{3{,}000} = \frac{5.5}{100}$$

$$x = \frac{5.5 \times 3{,}000}{100}$$

$$x = \$165 \text{ (interest amount)}$$

$$y = kx$$

$$I = (pr)t$$

$$I = 3{,}000 \times 0.055 \times 1$$

$$I = \$165 \text{ (interest amount)}$$

Gloria was playing a stock market game at school. She chose a stock that paid a simple interest of 4% per month. She invested $2,250 for 6 months. How much interest did Gloria make?

$$25\left(\frac{x}{\$2,250} = \frac{4}{100}\right)25$$

$\$2,250 \div 25 = \$90$ per month

$\$90 \times 6$ months $= \$540$

$y = kx$

$I = prt$

$I = \$2,250 \times 0.04 \times 6$

$I = \$540$

Concept Mastery

✓ Students are able to use proportional relationships to solve multistep ratio and percent problems, including simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

A link to helpful web resources can be found on page 147 of the full Level 7 document.

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